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A tyre-terrain interaction model for off-road vehicles

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The problem of off-road vehicle tyre - terrain interaction is difficult enough to be accurately solved. The previous work of many researchers pointed the terramechanics engineers into the right direction and a variety of methods to the study of tyre - terrain interaction has been developed over the last decades, they range from the entirely empirical to the highly theoretical. In this paper, a mathematical model based on mechanical spring mass-damper system was used to simulate the tyre- terrain interaction. A terrain mechanical model was developed to simulate the soil deformation under the effect of flexible tyre loading. A computer program using MATLAB/Simulink software was developed to solve the new model. The simulation shows that the new tyre - terrain model provides satisfactory results than the old one. Based on the obtained results, it is evident that the proposed model represents a step forward that enables better understanding of tyre - terrain interaction.

Key words: Tyre - terrain interaction, soil modeling, off-road vehicle.

INTRODUCTION

Nowadays, the requirements on off-road vehicles are rising steadily. The ideal vehicle has to provide excellent off-road capability with reasonable operation economics, offers a high customizability for each specific mission. To meet these demands, on the development side a lot of parameters` studies have to be carried out.

Tyre - soil interaction

Wheel-terrain interaction has been shown to play a critical role in rough-terrain vehicle mobility (Bekker, 1969; Wong, 1989). However, modeling and identification of such phenomena is often overlooked in the field of applications, possibly because methods for terrain parameter identification require costly and dedicated testing equipment. Finally, terrain parameter estimation employs vehicles in new ways for improved scientific understanding of planetary composition. Dubowsky et al. (2003) developed simplified nonlinear

force balance equations relating stresses at the rigid-wheel-terrain interface to externally applied force such as the wheel weight and torque and solved for the terrain parameters c and φ. The wheel sinkage can be computed from vision-based techniques that have been recently developed at Missushtties Institute of Technology.

Lach (1997) stated that the rolling resistance can only be determined indirectly by the sinkage and that the sinkage should be the first parameter to be controlled.

Janosi and Hanamoto (1961) introduce another parameter which is the soil shear strength, that the circumferential force depends upon but it is mainly influenced by the rolling resistance which depends on the pressure firmness.

Yong (1972) developed models for tyres that accounted for energy losses from wheel slippage, as well as soil failure. Brixius (1987) developed traction prediction equations for bias ply tyres. Zoz (1987) incorporated the models into a spreadsheet with an algorithm that accounted for weight transfer from the hitch loads.

Wood et al. (1988, 1990) determined the relationship between thrust and dynamic load for radial tyres in the soil bins at the National Soil Dynamics Laboratory.

Upadhyaya (1997) presented a semi-empirical method of predicting traction for radial ply tyres and have run

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tests to derive the empirical coefficients for the tyre and soil parameters of the model. The aim of Genga et al. (2007) research is to discuss appropriate interpretations of real modes and complex modes in structural vibration, so as to achieve an in-depth understanding of the damping effects in complex structures, such as pneumatic tyres.

Soil bearing capacity

The most simplified and least accurate assumption to determine the relationship between tyre sinkage "Z" and contact pressure "P" correspond to:

$$P = k\,Z \tag{1}$$

where k is the modulus of soil deformation.

Bekker stated that there are evidences to approve and reject that assumption. Also Bekker (1956) proposed a general equation including "n", the exponent dependent on the type of soil, assuming the relationship in the following form:

$$P = k.\,Z^n \tag{2}$$

Bekker (1960) introduced the exponent of deformation "n", used previously by Taylor formula for Civil Engineering and the pressure sinkage relationship as:

$$P = \left(\frac{K_c}{b} + K_\emptyset\right) z^n \tag{3}$$

where k_c cohesive modulus and k_Φ frictional modulus representing the soil stiffness constants dependents upon n, and b is the smaller dimension of a rectangular plate or radius of a circular one.

Reece (1965) noted that k_c and k_Φ parameters in Bekker's equation changed with plate width.

Wong (2001) stated that Reece formula is valid for homogeneous terrain. Bekker's pressure sinkage relationship and Reece's modification are independent of the sinkage rate. Pope (1969) showed that the rate of sinkage is an important factor in that relationship and suggestions of development studies were done in order to find a correlation between soil resistance and sinkage rate.

Benoit and Gotteland (2006) proposed a new model for vehicle mobility (N2M) and he developed an equation with four model parameters. Comparisons against Bekker's equation and experimental analysis showed that the model was good enough and more adequate to use when only two plates are used in the test.

Brooks and Iagnemma (2005) suggested a vibration-

based terrain classification as a novel sensing mode for identifying terrain class according to load-bearing surfaces that lie below a thin terrain layer of different composition. A good experimental classification results is obtained using even a simple classification algorithm.

Dubowsky and Lagnemma (2001) stated that it is important for a vehicle to sense changing terrain conditions, and modify its control strategies to ensure a yet safe operation during high-speed. He developed a terrain characterization and identification.

Finally it is concluded that dynamic modeling of tyre soil system is needed to improve the understanding related to the off road vehicle design.

SOIL MODELING

The soil model focuses on the efficient known factors that are critical defining mobility of an off-road vehicle, such as the soil density, and the pressure-sinkage relations. Such factors can provide the basis for the study of ride performance, handling, and mobility of the vehicle in off-road conditions.

Figure 1 presents a new mathematical model for the soil; it has two layers (soil and sub soil). The variables M_s, M_{ss}, k_s , c_s and K_{ss} denote the mass, stiffness and damping rate of the soil and sub soil, respectively. The variables Z_s and Z_{ss} are the displacements of the soil, and sub soil sinkage respectively. The variable "F" is denoted by the input force for the system. The linear dynamic equations are written as:

$$M_s\,\ddot{Z}_s = F - C_s\left(\dot{Z}_s - \dot{Z}_{ss}\right)$$
$$- K_s\left(Z_s - Z_{ss}\right) \tag{4}$$

$$M_{ss}\,\ddot{Z}_{ss} = C_s\left(\dot{Z}_s - \dot{Z}_{ss}\right) + K_s$$
$$\left(Z_s - Z_{ss}\right) - K_{ss}\left(Z_{ss}\right) \tag{5}$$

And the state space equation can be written as:

$$\dot{X} = A\,X + B\,u$$
$$And\,,Y = C\,X + D\,u$$

Equation 6 presents the state space equations for new soil model that is subjected to a ramp force input signal. The value of maximum force depends upon the pressure sinkage relationship used in this study from previous work data by Shaaban (1983). By running the MATLAB program and by some iteration, we can get the needed parameters of the tested soil. In another words; Bekker's standard valves of soil (n, k_c and k_Φ) has been replaced
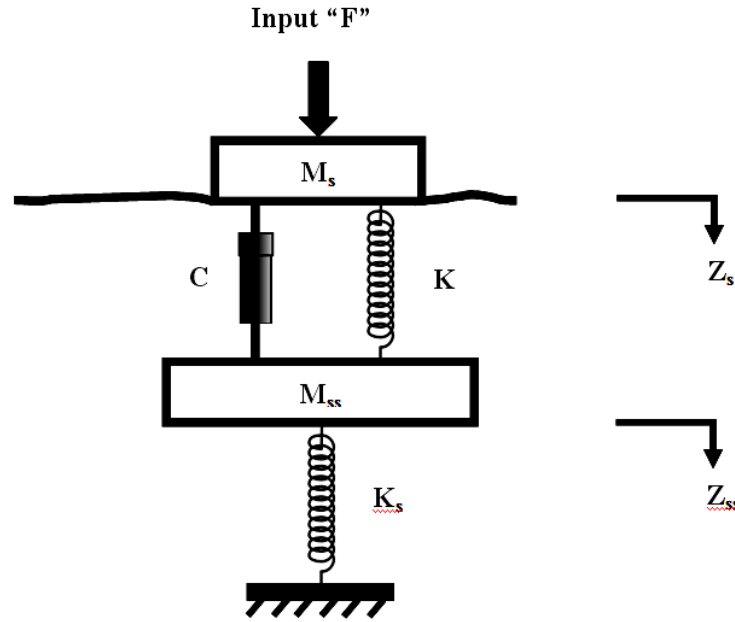
Figure 1. Mathematical model for the soil as spring – mass – damper system.

by the soil model parameters (M_s, M_{ss}, C_s, K_s and K_{ss}).

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \left(\dfrac{-K_s}{M_s}\right) & \left(\dfrac{K_s}{M_s}\right) & \left(\dfrac{-C_s}{M_s}\right) & \left(\dfrac{C_s}{M_s}\right) \\ \left(\dfrac{K_s}{M_{ss}}\right) & \left(\dfrac{-(K_s-K_{ss})}{M_{ss}}\right) & \left(\dfrac{C_s}{M_{ss}}\right) & \left(\dfrac{-C_s}{M_{ss}}\right) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [F] \tag{7}$$

Soil parameters

The non-linear relationship of pressure sinkage for the soil that was proposed by Bekker (1960) can be used to estimate the new soil model parameters (C_s, K_s and K_{ss}). By using the Terzaghi (1943) bearing capacity failure mechanism and assuming that the "active zone" introduces the soil mass, while the "passive and radial zone" represent the sub soil mass. Finally the mass of the soils has been calculated using modern computer aided design software (Figure 2). Figures 3 and 4 show

the soil model results compared and validated by the previous experimental work by Shaaban (1983) with Bekker's equation.

TYRE SOIL MODEL

Figure 5 shows the new model has the capability to calculate the sinkage and deflection of a tested tyre for specific tyre inflation pressure running on different type of terrain. The tyre soil model variables M_w, M_s and M_{ss} denote the mass of the wheel, soil and sub soil, respectively. The variables K_t, K_s, K_{ss}, C_t, and C_s denote the spring stiffness and damping coefficient tyre, soil, and sub soil, respectively. The variables Z_w, Z_s and Z_{ss} are the displacement of wheel, soil and sub soil, respectively. The variable "F" denotes the input force to the system.

The tyre sinkage and deflection have been recorded at specific tyre inflation pressure. For this passive model, the tyre inflation pressure can be changed by changing the tyre stiffness and damping coefficient. Furthermore, types of soil can be widely ranged in this model accordingly to the identified soil parameters outcomes from soil modeling.

The linear dynamic equations for the tyre soil model are:

For *wheel* mass;

$$M_w \ddot{Z}_w = F - C_t \left(\dot{Z}_w - \dot{Z}_s \right) - K_t \left(Z_w - Z_s \right) \tag{8}$$
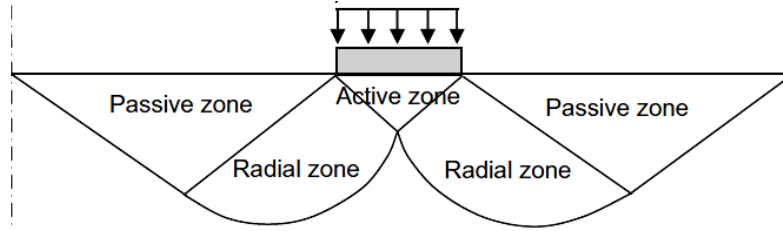
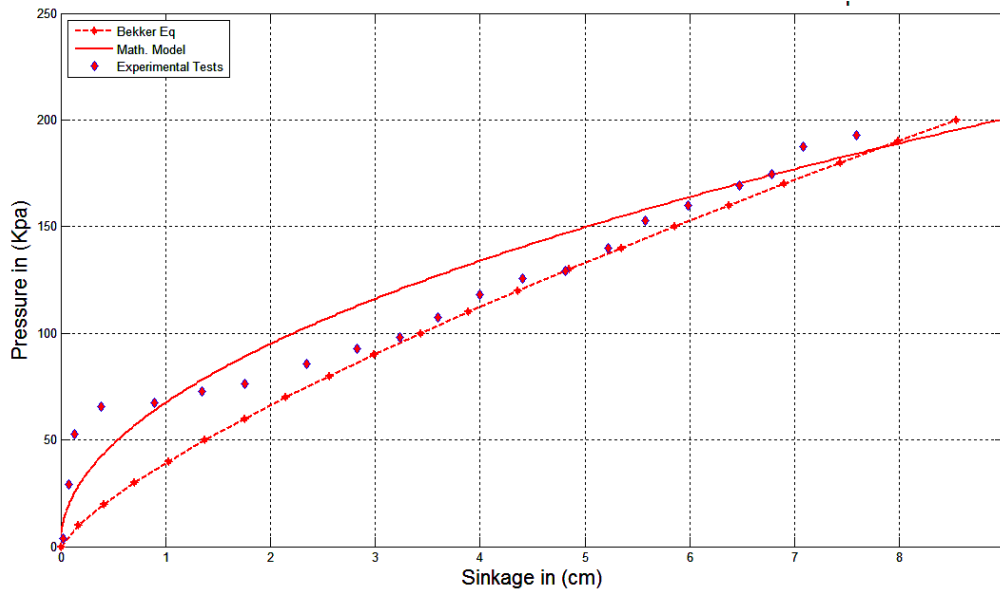**Figure 2.** Terzaghi failure mechanism under a uniform load footing (Junhwan and Jongwan, 2009).

Figure 3. Soil mathematical model validation for a soil density 16.4 (kN/m^3) (Shaaban, 1983).

For *soil* mass;

$$M_s \, \ddot{Z}_s = C_t \left(\dot{Z}_w - \dot{Z}_s \right) + K_t \left(Z_w - Z_s \right) - C_s \left(\dot{Z}_s - \dot{Z}_{ss} \right) - K_s \left(Z_s - Z_{ss} \right) \tag{9}$$

For *sub soil* mass;

$$M_{ss} \, \ddot{Z}_{ss} = C_s \left(\dot{Z}_s - \dot{Z}_{ss} \right) + K_s \left(Z_s - Z_{ss} \right) - K_{ss} \left(Z_{ss} \right) \tag{10}$$

Active model

Active tyre - soil system has been designed to improve the vehicle performance by adjusting the optimum tyre inflation pressure by using a controlled actuator (Figure 6). The presented control concept has been established by a force f_a, applied by an actuator. The actuator is assumed to be placed between the wheel mass (M_w) and soil mass (M_s) which can exert a force f_a relative to the change of inflation pressure in the tyre.

The active tyre soil model linear dynamic equations are written as:

For *wheel* mass;

$$M_w \, \ddot{Z}_w = F - C_t \left(\dot{Z}_w - \dot{Z}_s \right) - K_t \left(Z_w - Z_s \right) - f_a \tag{11}$$

For soil mass;

$$M_s \, \ddot{Z}_s = C_t \left(\dot{Z}_w - \dot{Z}_s \right) + K_t \left(Z_w - Z_s \right) - C_s \left(\dot{Z}_s - \dot{Z}_{ss} \right) - K_s \left(Z_s - Z_{ss} \right) + f_a \tag{12}$$
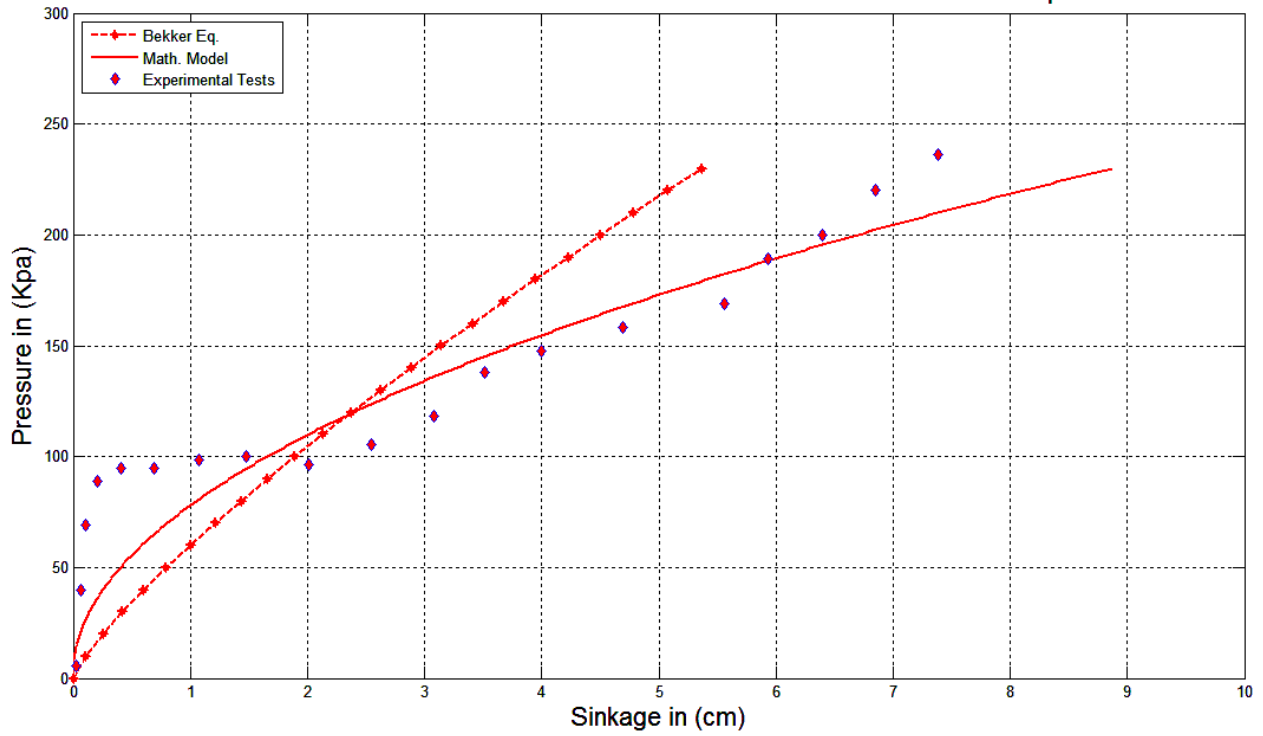
Figure 4. Soil mathematical model validation for a soil density 16.9 (kN/m^3) (Shaaban, 1983).
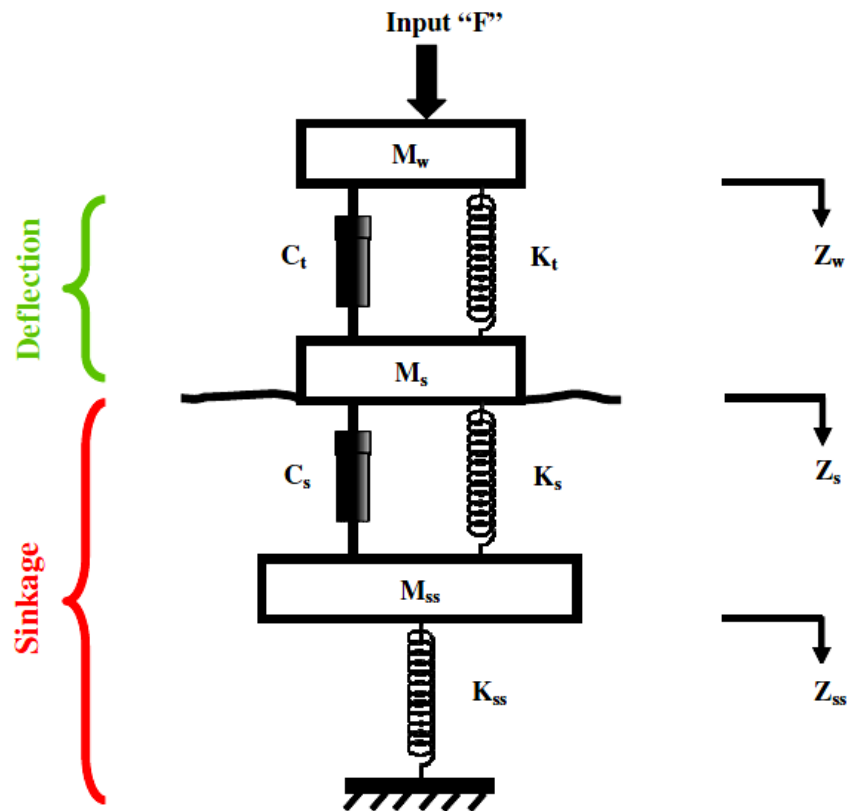

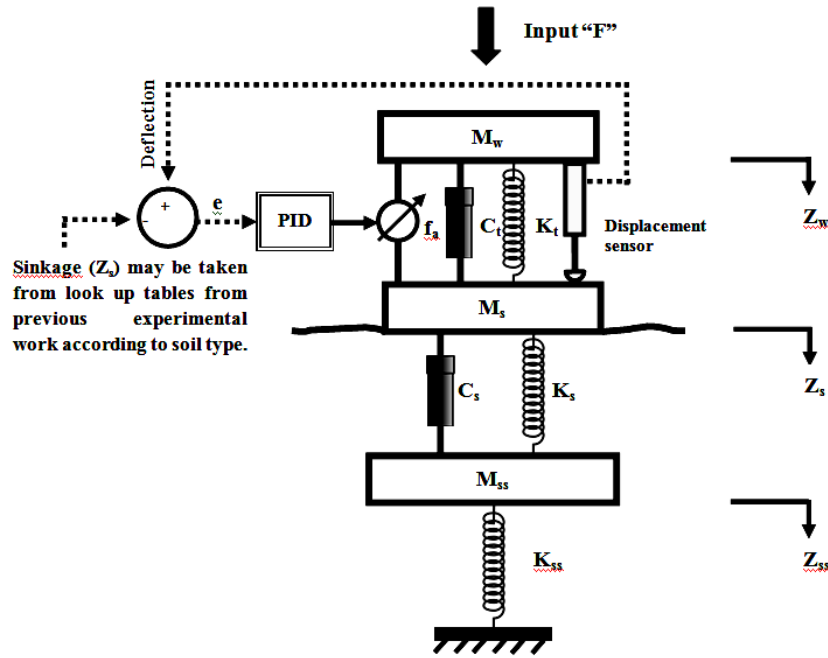**Figure 5.** Tyre soil model as spring – mass – damper system.

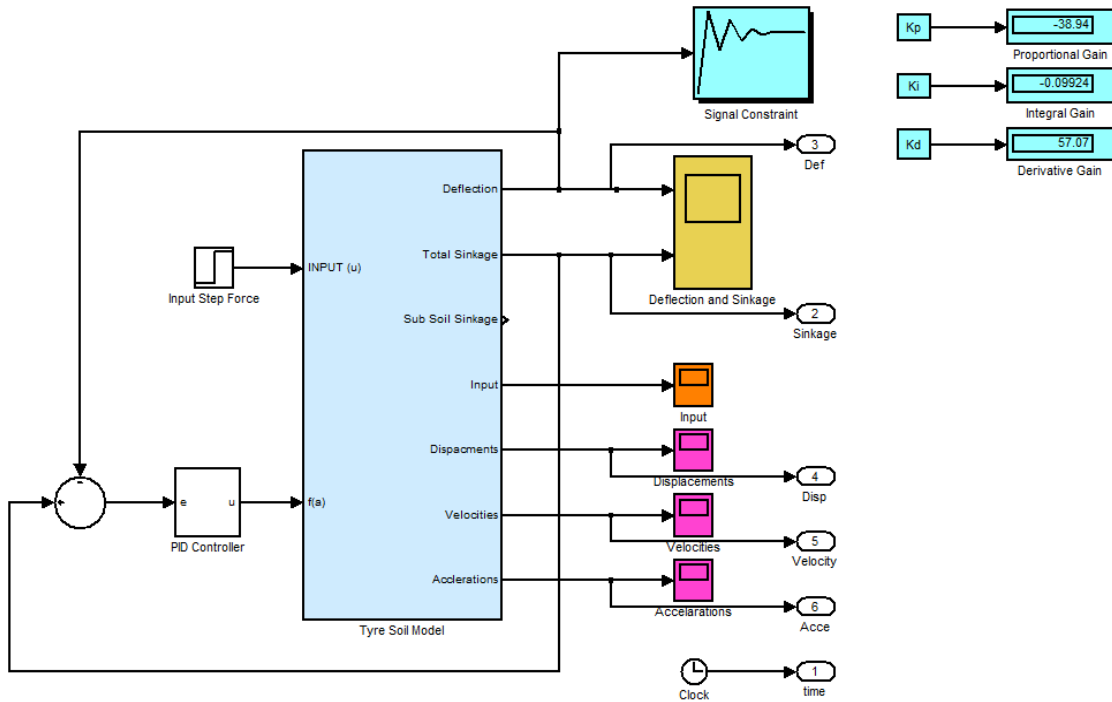**Figure 6.** Active tyre soil model with PID controller.

Figure 7. Simulation of active tyre soil model by using MATLAB/Simulink Program.

For *sub soil* mass;

$$M_{ss} \ddot{Z}_{ss} = C_s \left(\dot{Z}_s - \dot{Z}_{ss} \right) + K_s \left(Z_s - Z_{ss} \right) - K_{ss} \left(Z_{ss} \right) \quad (13)$$

The model was built in Simulink based on Equations (11) to (13) as shown in Figure 7. The actuator force is controlled by the PID (proportional-integral-derivative)
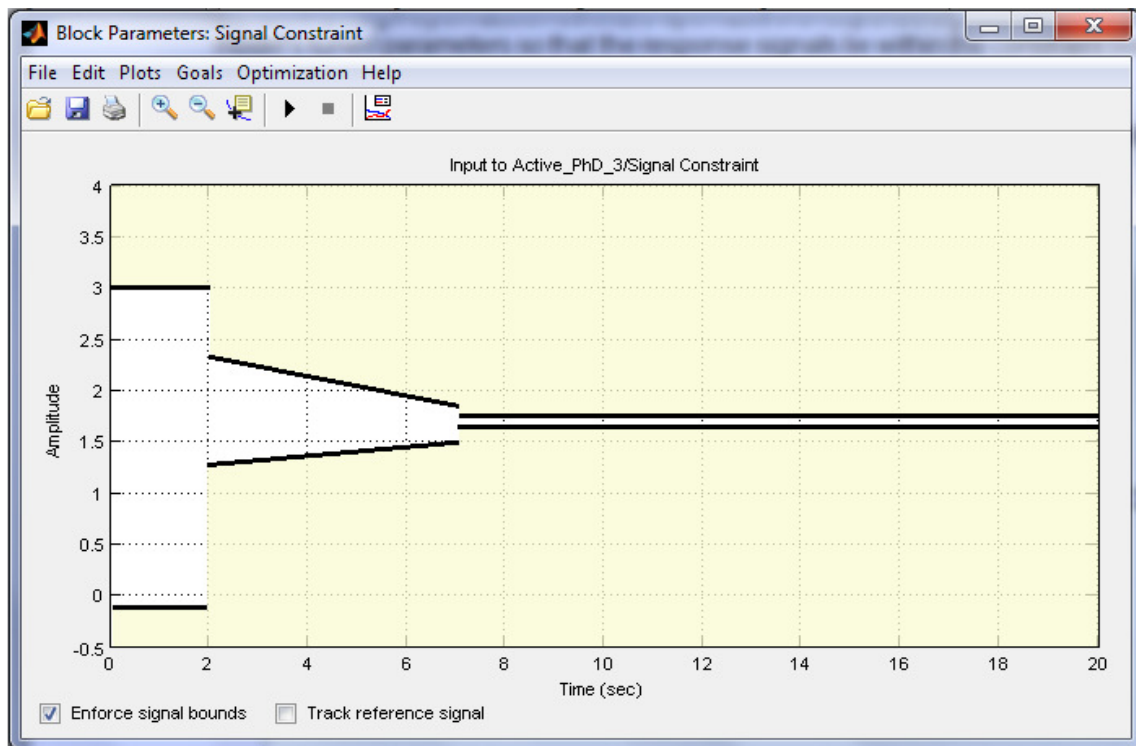
Figure 8. The designed segment constraints for tyre deflection signal.

controller which involves a feedback loop. The "error" signal (difference between sinkage and the deflection of the tyre) is used to adjust the input to the system in order to bring the process' measured value back to the desired. PID gains (K_p, K_i, and K_p) are the parameters to be adjusted to tune the controller for the required performance of the system.

PID CONTROLLER OPTIMIZATION

The optimization has been carried out using MATLAB/Simulink constraint optimization toolbox. The tyre deflection signal has been measured in the Simulink active tyre soil model to optimize the model response. Simulink design optimization tunes parameters (controller gains K_p, K_i, and K_d) in the tyre soil model to meet specified constraints, as shown in Figure 8. During the optimization, intermediate responses are also plotted and finally the best response has been plotted and selected.

PARAMETRIC STUDY

The performance of the off-road vehicle is influenced by a number of factors, such as terrain type, vehicle speed, vehicle geometry, and vehicle system parameters associated with loading conditions and effect of tyre inflation pressure. A better understanding of these factors is achieved through a parametric study for the available parameters in the tyre soil model.

The objective of the parametric study presented is to demonstrate the influence of variations in the tyre soil model parameters on the model behavior. The parameters considered for variations are: Tyre stiffness and soil type which includes all the soil parameters (Figures 9 and 10).

Based on this parametric study, the following observations are drawn:

1. As the tyre stiffness increases, tyre deflection decreases and tyre sinkage increases; it means that the tyre tends to be a rigid tyre. Decreasing the tyre stiffness to half its original value more tyre deflection and lower tyre sinkage occur, this is shown in Figure 9.
2. A reduction in soil stiffness (that is, changing from dense soil to loose soil) implies that tyre sinkage gets extremely larger in the loose soil than in the dense soil and the tyre deflection clearly reduced as shown in Figure 10.

In this parametric study, it was found that the response of the tyre deflection and the sinkage of the tyre soil model as described previously vary according to by variation of tyre stiffness and soil rigidity. Based on the available data in the literature review, it was found that the outcome
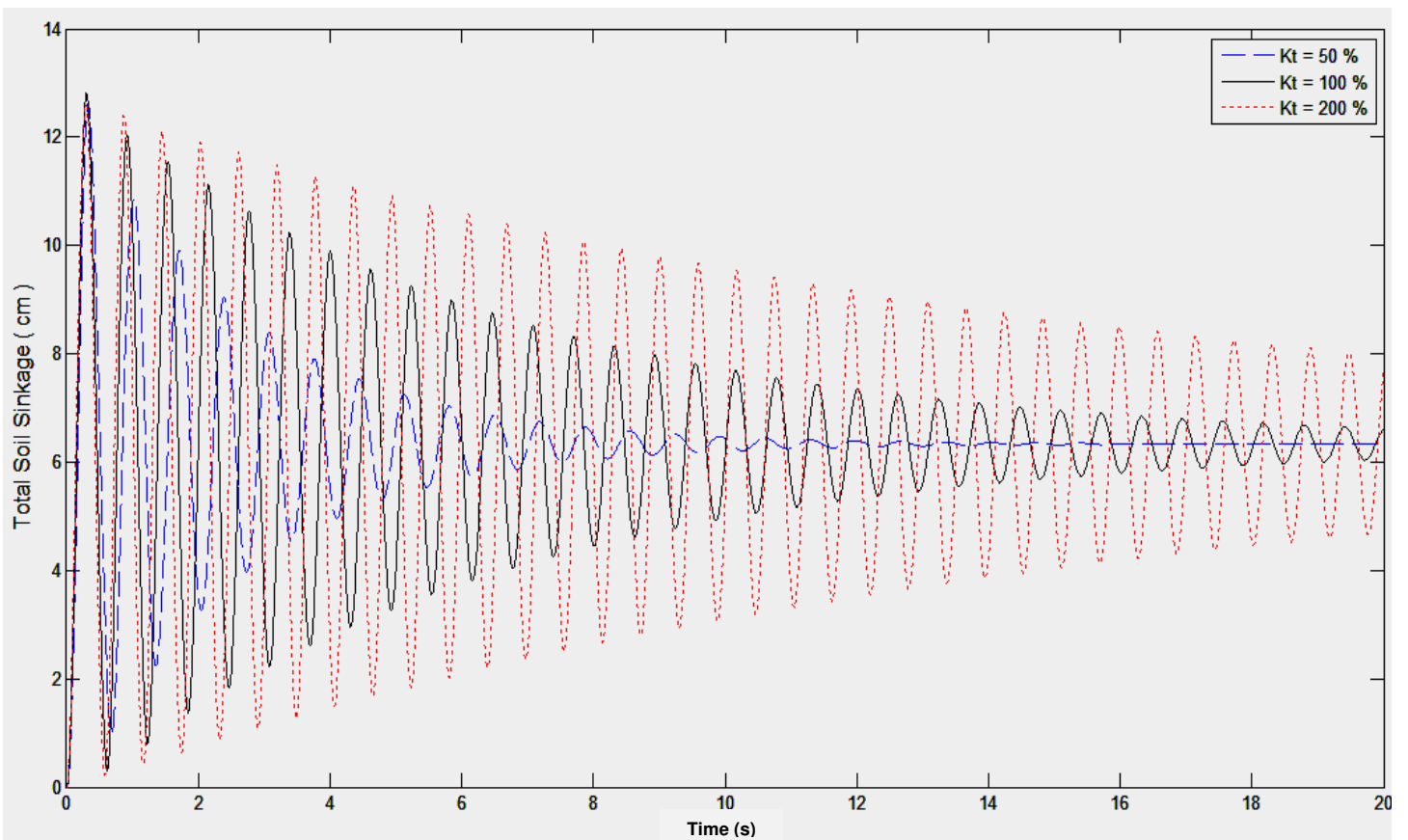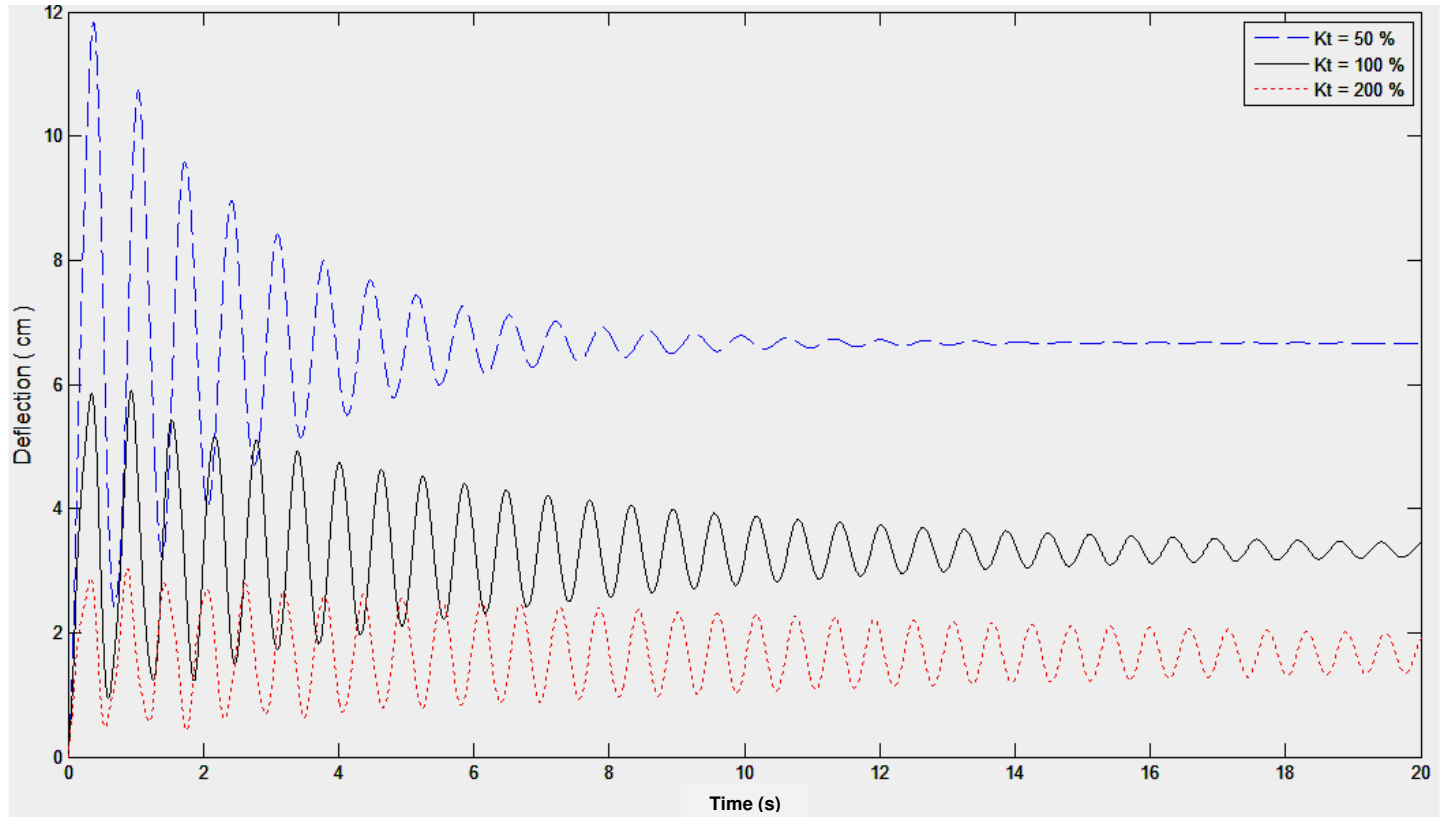
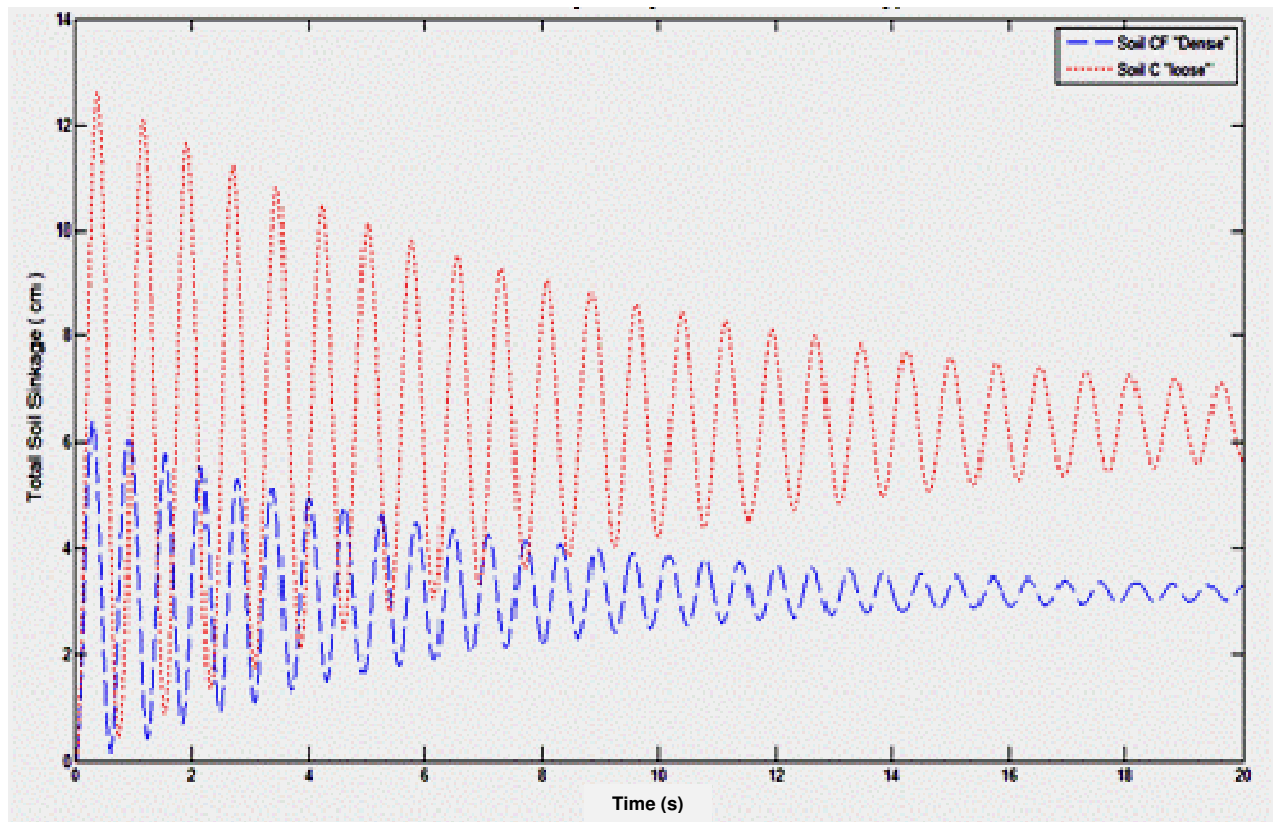**Figure 9.** Simulation results for tyre deflection and tyre sinkage at different tyre stiffness.
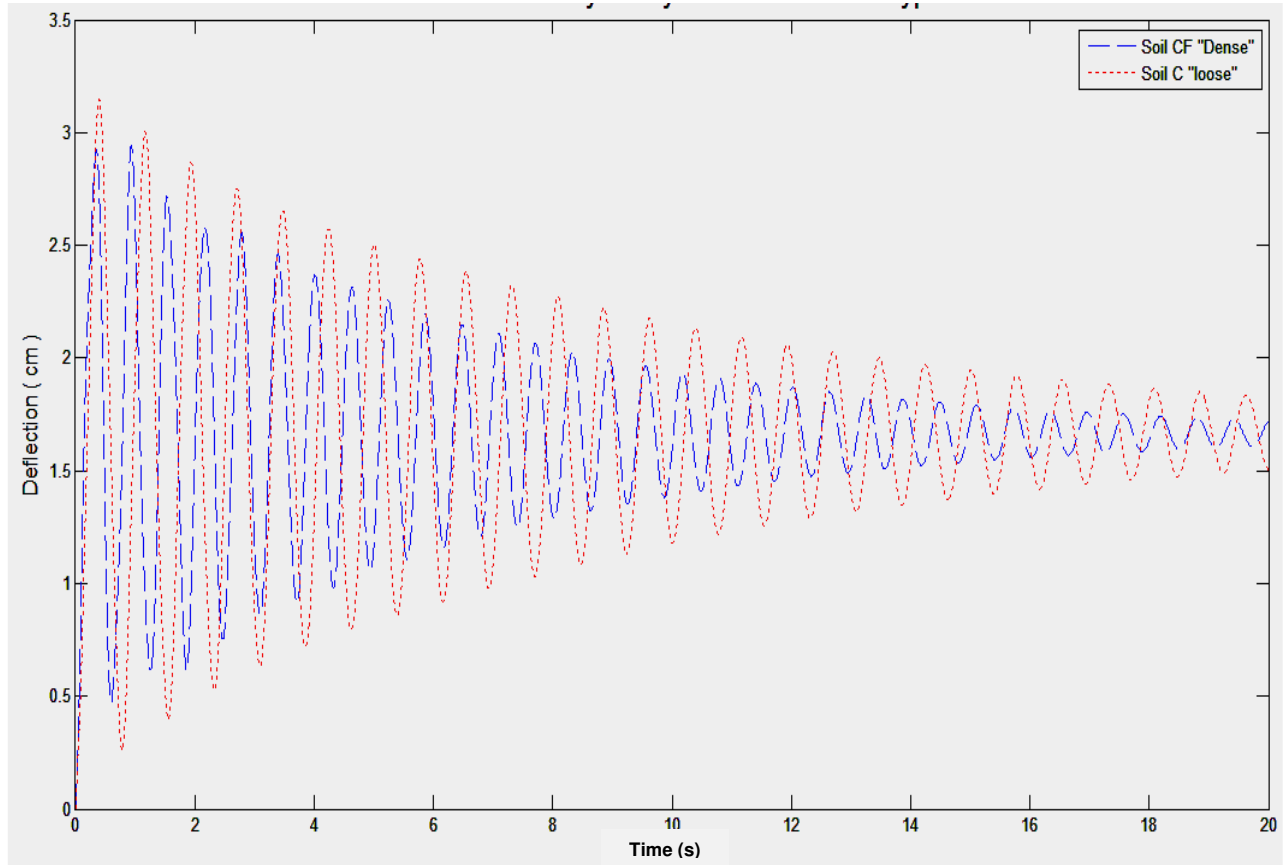
Figure 10. Simulation results for tyre deflection and tyre sinkage at two different soils (loose and dense).
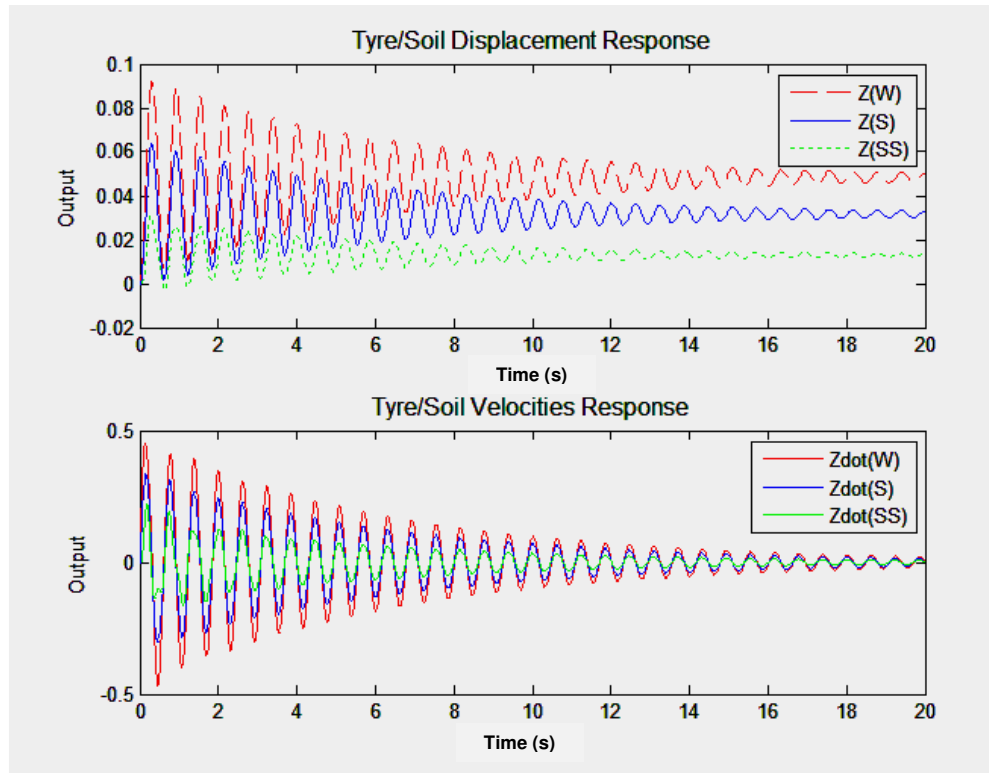
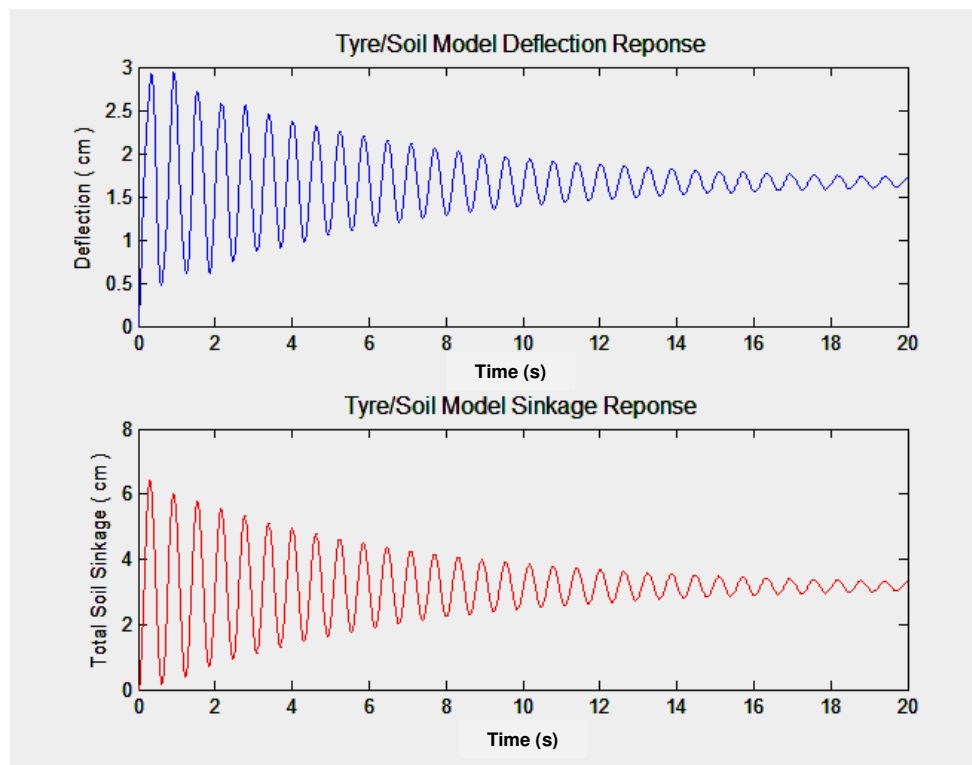**Figure 11.** Passive tyre soil model response for displacements and velocities to a step input disturbance

Figure 12. Passive tyre soil model response for tyre deflection and sinkage to a step input disturbance.
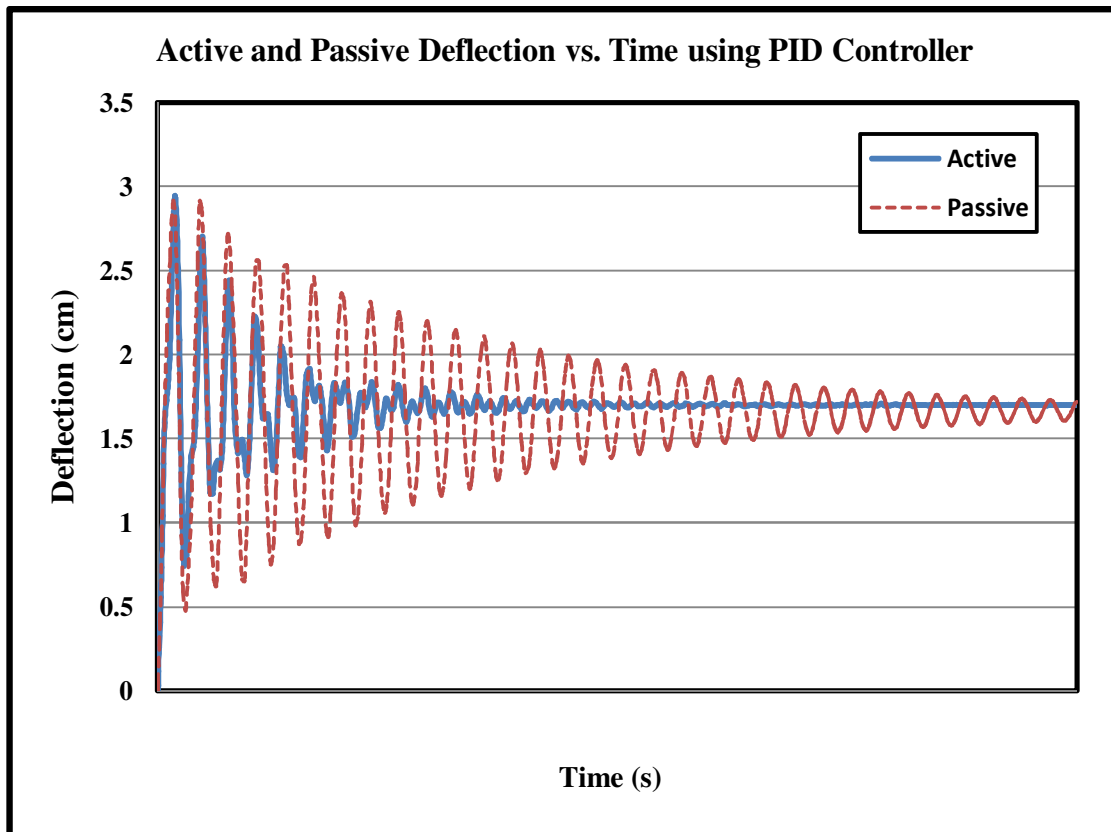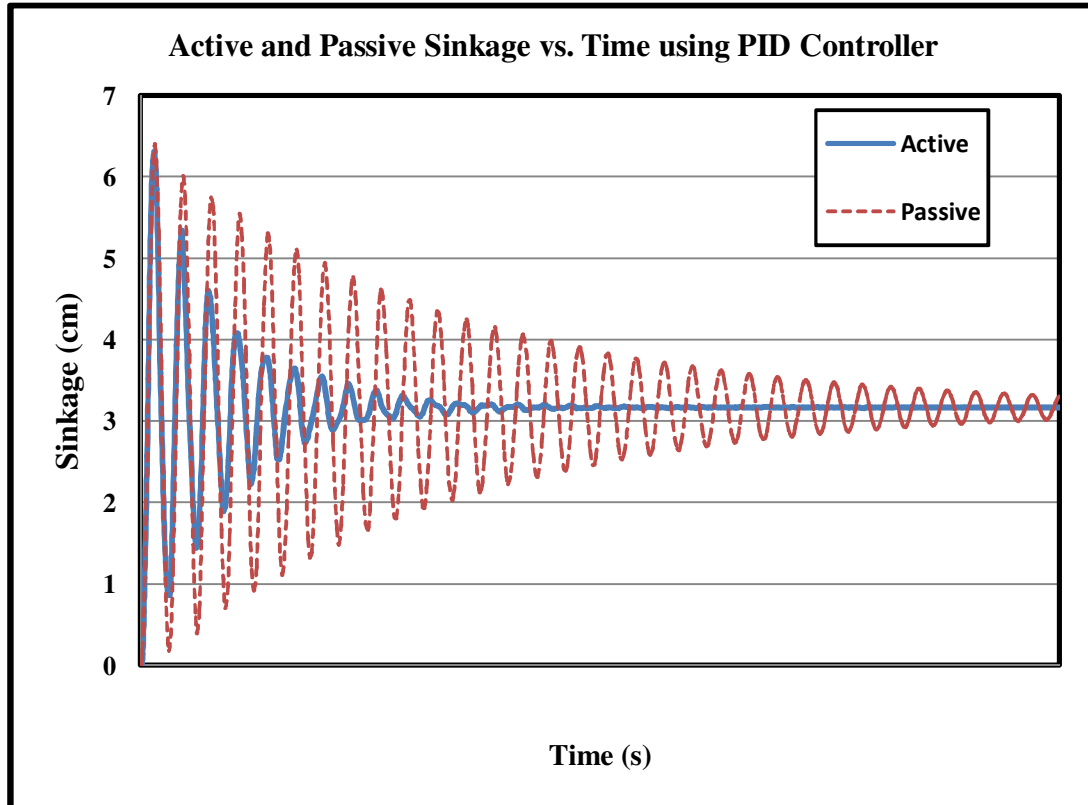
Figure 13. Comparison between active and passive response for tyre sinkage and deflection.
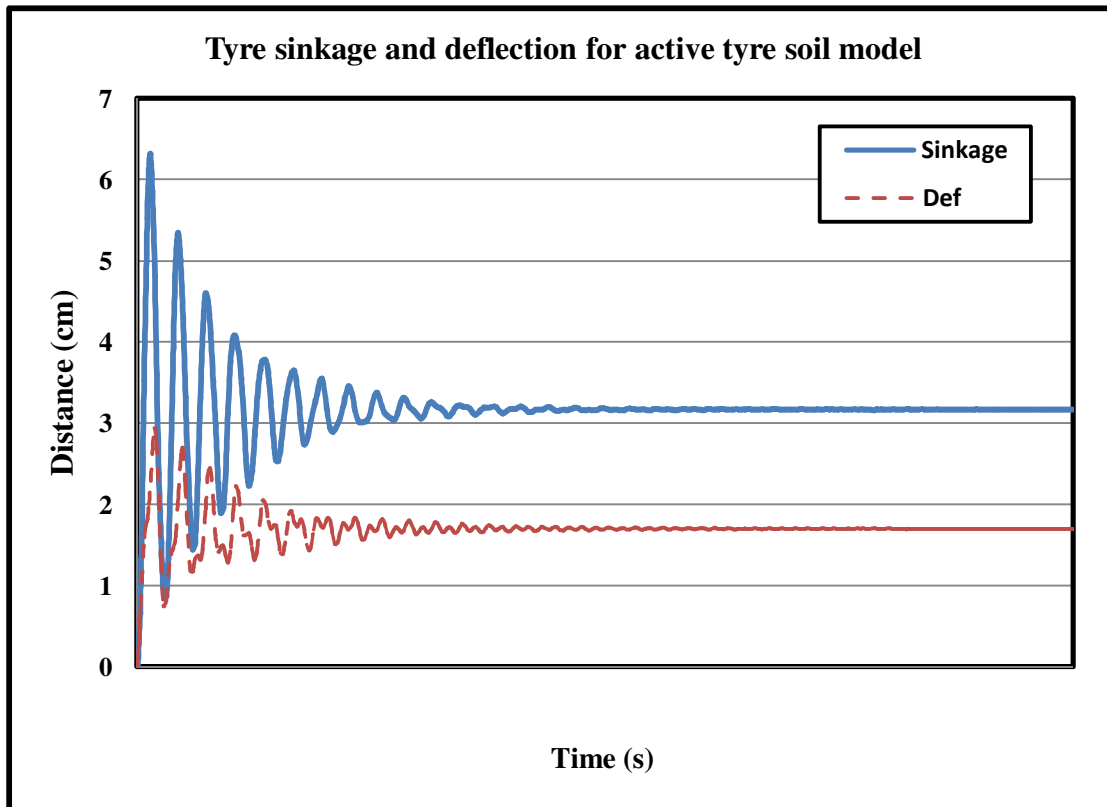
Figure 14. Active tyre soil model response for tyre sinkage and deflection.

results from this parametric study are valuable and satisfactory.

RESULTS

Comparison of the results related to the two systems (passive and active) has been performed. Figures 11 and 12 present the passive tyre soil model response subjected to the step input disturbance. The upper part in Figure 11 is the wheel, soil, and sub soil displacements response in meter, while the bottom part shows the wheel, soil, and sub soil velocities in meter per second. Figure 12 shows the tyre deflection and sinkage in centimeters.

Comparison between active and passive system simulation results are graphically shown in Figure 13. The solid line indicates the response of the active system for tyre sinkage and deflection respectively, while the dotted line indicates the response of the passive system for tyre sinkage and deflection respectively. In Figure 14, the tyre deflection and sinkage response for the active tyre soil model with a tuned PID controller is presented. It is obvious that the active system produces the best performance compared to the passive system for the introduced disturbance. This shows that the active

system is more effective and more rapid.

Conclusions

This paper presents an algorithm for adapting the tyre inflation pressure of off road vehicles operating on rough terrain to reach tyre floatation. The algorithm accounts for dynamic effects of tyre sinkage and tyre deflection in the vehicle/terrain interaction. Extensive simulation and experimental results demonstrate the method effectiveness. A time-domain Matlab simulation passive model was developed to study the "tyre soil" dynamic performance and to simulate the floatation pressure adaption.

The parametric study of the new passive model showed that by increasing tyre stiffness, tyre deflection decreases and tyre sinkage increase, while as by soil stiffness decreasing tyre sinkage becomes extremely larger and its deflection clearly reduced. The active tyre soil model has been designed to simulate the adjustment of optimum tyre pressure by a controlled actuator that applies a force (f_a) relative to the change of inflation pressure. The controller gains (K_p, K_i, K_d) are optimized using a Simulink program according to the current model parameters.

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