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*Full Length Research Paper*

# Kinematic model of three wheeled mobile robot

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**In this paper, the behavior of wheeled mobile robots (WMR) has been analyzed. These robots ride on a system of wheels and axles, some of which may be steerable or driven. For WMRs, there are many wheels and axle configuration that have been used. The present work outlined for five categories of WMRs. The ultimate objective of this paper is to investigate the complete description of the control theory of such robots and its maneuverability. Equations are modeled to describe the rigid body motions that arise from rolling trajectories based on the geometrical constraints of these wheels. Finally this analysis is applied to various three wheeled mobile robots.**

**Key words:** Wheeled mobile robot, kinematic analysis, degree of maneuverability.

## INTRODUCTION

A mobile robot is a combination of various hardware and software components in order to move in its free space. It is a collection of subsystems as shown in Figure 1; *Locomotion*: it enables how the robot to move unbounded throughout its environment. *Sensing*: How the robot measures properties of itself and its environment. *Control*: How the robot generate physical actions. *Reasoning*: How the robot maps measurements into actions; *Communication*: How the robots communicate with each other or with an outside operator. But there are a large variety of possible ways (Xiaodong and Shugen, 2010) to move, and so the selection of a robot's approach to locomotion is an important aspect of mobile robot design. Understanding mobile robot motion starts with understanding constraints placed on the robots mobility. Owing these limitations, mobile robots generally locomote either using wheeled mechanisms, a well-known human technology for vehicles, or using a small number of articulated legs (Júlia and Federico, 2009), the simplest of the biological approaches to locomotion. In general, legged locomotion requires higher degrees of freedom and therefore greater mechanical complexity than wheeled locomotion. Wheels, in addition to being simple, are extremely well suited to flat ground.

Alexandery and Maddocks (1988) modeled a wheeled mobile robot as a planar rigid body that rides on an arbitrary number of wheels. And they developed a relationship between the rigid body motion of the robot and the steering and drive rates of wheels. The structure of the kinematic and dynamic models has been analyzed by Guy et al. (1992), for various wheeled mobile robots. Those wheel types, ome of the researchers have been dealt with some types of wheels. Wheel architecture has been developed by Kim et al. (2003), for the holonomic mobile platform in order to provide omni-directional motions by three individually driven and steered wheels. Jae et al. (2007) investigated the kinematics of a mobile robot with the proposed double-wheel-type active caster, which is developed as a distributed actuation module and can endow objects with mobility on the planar workspace. For a mobile robot equipped with N Swedish wheels, Giovanni (2009), has been described its kinematic modeling, singularity analysis, and motion control. Vrunda et al. (2010) have analysed on spherical wheeled mobile robot for feasible path planning and feedback control algorithms. Nilanjan and Ashitava (2004) modeled the wheels as a torus and proposed the use of a passive joint allowing a lateral degree of freedom for kinematic analysis of a wheeled mobile robot (WMR) moving on uneven terrain. A University of Minnesota's Scout is a small cylindrical robot has been modeled by Sascha et al.

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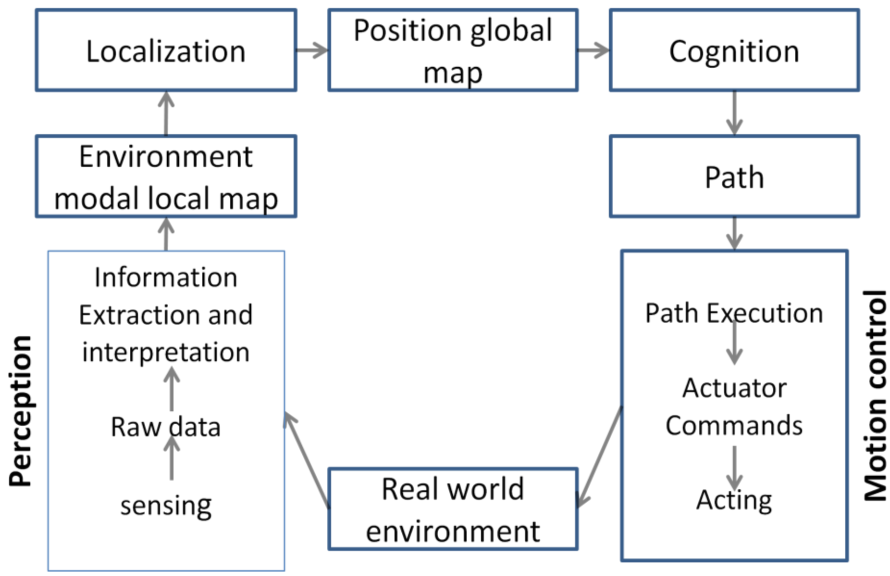


**Figure 1.** Reference control scheme for mobile robot systems.

(2006), which is capable of rolling and jumping for motion prediction. With the help of some interactive tools or algorithms, the movement of a mobile robot can be controlled. Guzm´ana et al. (2008), proposed an interactive tool for solving mobile robot motion problems by understanding several well-known algorithms and techniques. Johann (1995) introduced a new concept in the control and kinematic design of multi degree of freedom (MDOF) mobile robots. To control wheel slippage compliant, linkage is provided compliance between the drive wheels or drive axles of a vehicle. A cross-coupling control algorithm has been developed by Johann and Yoram (1986), that guarantees a zero steady-state orientation error and a stability analysis of the control system is presented.

The present paper deals with the kinematic model for general wheel mobile robots (WMR), while the robots are equipping with various types of wheels. Our purpose is to point out the structural properties of the kinematic models for a WMR. By introducing the concepts of degree of mobility and of degree of steeribility, we show that the variety of possible robot constructions and wheel configurations. The set of WMR can be partitioned in five classes and this analysis has been carried out in 'Kinematic models for various wheeled mobile robots'.

## KINEMATIC MODELS FOR VARIOUS WHEELED MOBILE ROBOTS

### Representation of a robot position

A wheeled mobile robot is a wheeled vehicle which is capable of an autonomous motion because it is equipped with motors that are driven by an embarked computer. Throughout this analysis, we model the robot as a rigid body on wheels, operating on a horizontal plane. The total dimensionality of this robot chassis on the plane is three, two for position in the plane and one for orientation along the vertical axis, which is orthogonal to the plane.

In order to specify the position of the robot on the plane, we establish a relationship between the global reference frame of the plane and the local reference frame of the robot, as in Figure 2. The axes of an arbitrary inertial basis on the plane as the global reference frame from some origin $O$:$\{X_I, Y_I\}$ to specify the position of the robot. Consider a point $P$ on the robot chassis as its position reference point. The basis $\{X_R, Y_R\}$ defines two axes relative to $P$ on the robot chassis and is thus the robot's local reference frame. The position of $P$ in the global reference frame is specified by coordinates $x$ and $y$, and the angular difference between the global and local reference frames is given by $\theta$.

Therefore the robot position:

$$\xi_I = [x \quad y \quad \theta]^T \tag{1}$$

And mapping is accomplished using the orthogonal rotation matrix:

$$R(\theta) = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

From Equation (2), we know that we can compute the robot's motion in the global reference frame from motion in its local reference frame:

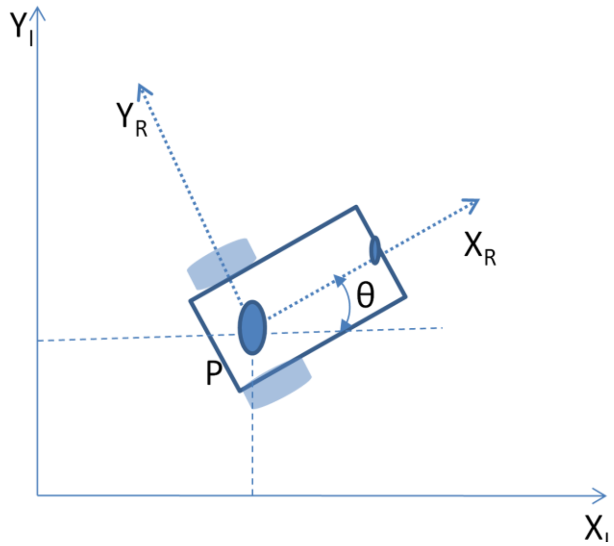


**Figure 2.** The global reference plane and the robot local reference frame.

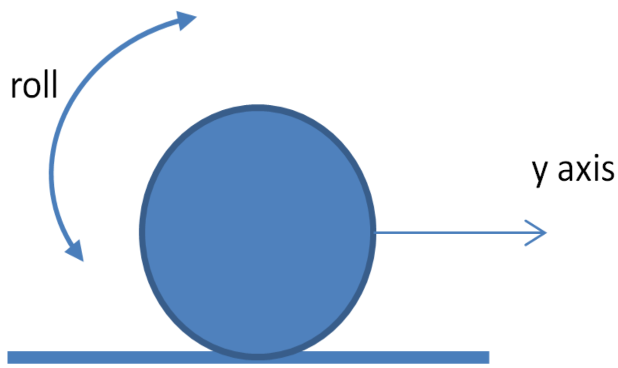


**Figure 3.** Rolling motion.

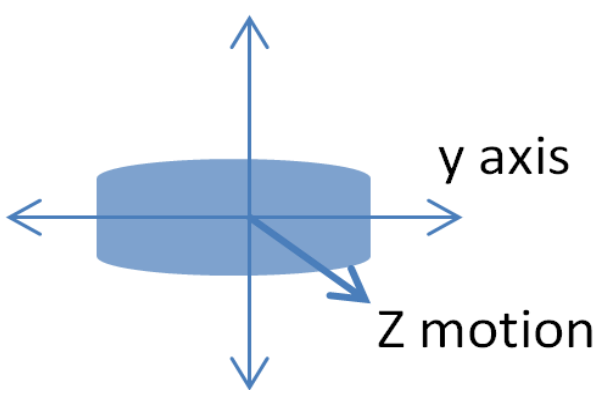


**Figure 4.** Lateral slip.

$$\xi_R = R(\theta)\xi_I \text{ and } \dot{\xi}_R = R(\theta)\dot{\xi}_I \quad (3)$$

$$\dot{\xi}_I = R(\theta)^{-1}\dot{\xi}_R \quad (4)$$

**Kinematic constraints of various wheel configurations**

While a wheeled robot is in movement, we present two constraints for every wheel type. The first constraint enforces the concept of rolling contact as represented in Figure 3; wheel must roll when motion takes place in the appropriate direction. The second constraint enforces the concept of no lateral slippage, that the wheel must not slide orthogonal to the wheel plane as shown in Figure 4. The first step to a kinematic model of the robot is to express constraints on the motions of individual wheels. The motions of individual wheels can be later combined to compute the motion of the robot as a whole.

We will consider the four basic wheel types:

1. Fixed standard wheel;
2. Steered standard wheel;
3. Swedish wheel;
4. Spherical wheels
5. Castor wheel

We assume that, during the motion, the plane of the wheel always remains vertical and there is no sliding at the single point of contact between the wheel and the ground plane (The wheel undergoes motion only under conditions of pure rolling and rotation about the vertical axis through the contact point).

***Fixed standard wheel***

The fixed standard wheel has no vertical axis of rotation for steering. Its angle to the chassis is thus fixed, and it is limited to motion back and forth along the wheel plane and rotation around its contact point with the ground plane. Figure 5 depicts a fixed standard wheel and indicates its position pose relative to the robot's local reference frame. The position of robot chassis is expressed in polar coordinates by distance l and angle α. The angle of the wheel plane relative to the chassis is denoted by β. The wheel, which has radius r, can spin over time, and so its rotational position around its horizontal axle is a function of time t: φ(t).

The rolling constraint for this wheel enforces that all motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel spin so that there is pure rolling at the contact point:

$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad (-l)\cos\beta] *$$

$$R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \quad (5)$$

The sliding constraint for this wheel enforces that the

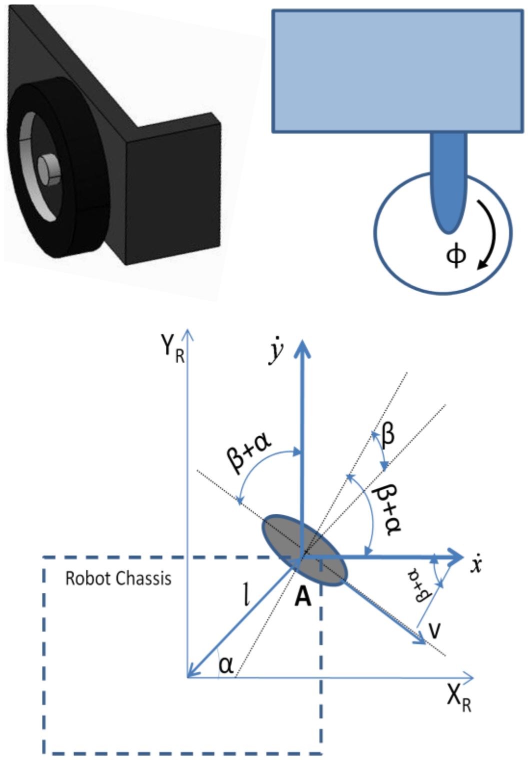


**Figure 5.** Fixed standard wheel and its parameters.

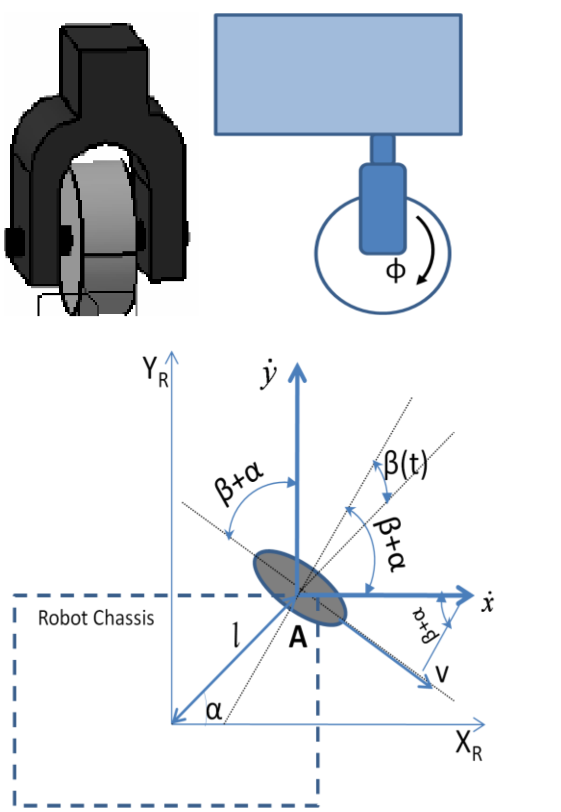


**Figure 6.** Steerable standard wheel and its parameters.

component of the wheel's motion orthogonal to the wheel plane must be zero:

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\,sin\beta]R(\theta)\dot{\xi}_I = 0 \tag{6}$$

***Steered standard wheel***

The steered standard wheel differs from the fixed standard wheel only in that there is an additional degree of freedom: The wheel may rotate around a vertical axis passing through the center of the wheel and the ground contact point. The orientation of the wheel to the robot chassis is no longer a single fixed value $\beta$, but instead varies as a function of time: $\beta(t)$ .

The rolling and sliding constraints for the steered standard wheel shown in Figure 6:

$$[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad (-l)cos\beta]*$$

$$R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \tag{7}$$

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad lsin\beta]R(\theta)\dot{\xi}_I = 0 \tag{8}$$

***Swedish wheel***

Swedish wheels have no vertical axis of rotation, yet are able to move omnidirectionally like the castor wheel. This is possible by adding a degree of freedom to the fixed standard wheel. Swedish wheels consist of a fixed standard wheel with rollers attached to the wheel perimeter with axes that are antiparallel to the main axis of the fixed wheel component. The exact angle γ between the roller axes and the main axis can vary, as shown in Figure 7.

The motion constraint that is derived looks identical to the formula is modified by adding γ such that the effective direction along which the rolling constraint holds is along this zero component rather than along the wheel plane:

$$[\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad (-l)\cos(\beta+\gamma)]*R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \tag{9}$$

Orthogonal to this direction, the motion is not constrained because of the free rotation $\dot{\varphi}_{sw}$ of the small rollers.

$$[\sin(\alpha+\beta+\gamma) \quad \cos(\alpha+\beta+\gamma) \quad (l)\sin(\beta+\gamma)]*R(\theta)\dot{\xi}_I - r\dot{\varphi}sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0 \tag{10}$$

Consider $\gamma = 0$, this represents the Swedish 90-degree wheel. In this case, the zero component of velocity is in line with the wheel plane and so Equation (9) reduces exactly to Equation (4), the fixed standard wheel rolling constraint. But because of the rollers, there is no sliding

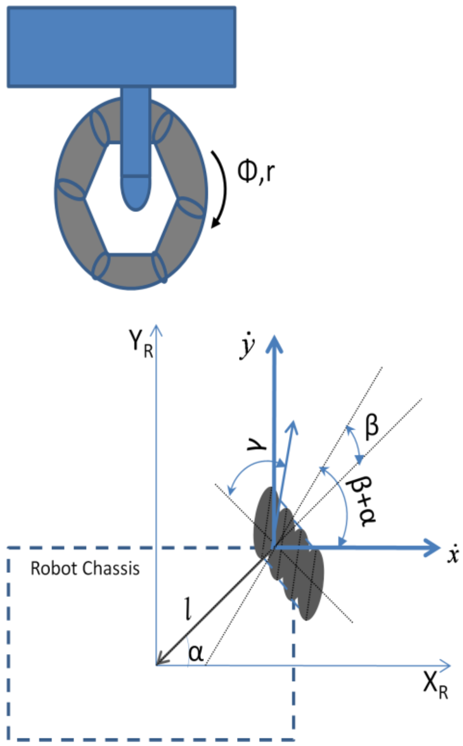


**Figure 7.** Swedish wheel and its parameters.

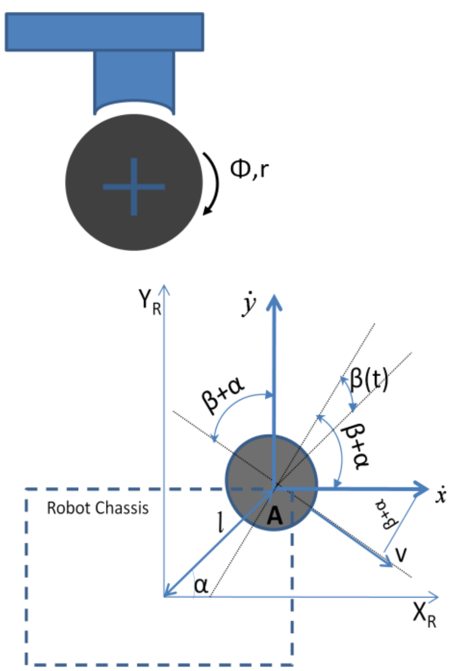


**Figure 8.** Spherical wheel and its parameters.

constraint orthogonal to the wheel plane. At the other extreme $\gamma = \frac{\pi}{2}$, the rollers have axes of rotation that are parallel to the main wheel axis of rotation. For $\gamma = \frac{\pi}{2}$ in Equation (9) the result is the fixed standard wheel sliding constraint, Equation (5).

### *Spherical wheel*

A ball or spherical wheel, places no direct constraints on motion (Figure 8). Such a mechanism has no principal axis of rotation, and therefore no appropriate rolling or sliding constraints exist. Therefore Equation (11) simply describes the roll rate of the ball in the direction of motion $v_A$ of point A of the robot.

$$[\sin(\alpha+\beta) - \cos(\alpha+\beta)(-l)\cos\beta]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \quad (11)$$

By definition, the wheel rotation orthogonal to this direction is zero.

$$[\sin(\alpha+\beta)\cos(\alpha+\beta)l\ sin\beta]R(\theta)\dot{\xi}_I = 0 \quad (12)$$

### *Castor wheel*

Castor wheels are able to steer around a vertical axis. However, unlike the steered standard wheel, the vertical axis of rotation in a castor wheel does not pass through the ground contact point. Figure 9 depicts a castor wheel, demonstrating that formal specification of the castor wheel's position requires an additional parameter which is a rigid rod of fixed length connected to wheel.

For the caster wheel, the rolling constraint is identical to equation because the offset axis plays no role during motion that is aligned with the wheel plane:

$$[\sin(\alpha+\beta) - \cos(\alpha+\beta)(-l)cos\beta]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0 \quad (13)$$

Because of the offset ground contact point relative to A, the constraint that there be zero lateral movement would be wrong. Instead, the constraint is much like a rolling constraint, in that appropriate rotation of the vertical axis must take place:

$$[\cos(\alpha+\beta)\ \sin(\alpha+\beta)d + l\ sin\beta]R(\theta)\dot{\xi}_I + d\dot{\beta} = 0 \quad (14)$$

### Kinematic constraints for a robot

We now consider a general mobile robot, equipped with *N* wheels of the five above described categories. We use the five following subscripts to identify quantities relative to these classes: *f* for fixed wheels, s for steerable standard wheels, sw for Swedish wheels, sp for spherical wheel and c for castor wheels. The numbers of wheels for each type are denoted $N_f$, $N_s$, $N_c$, $N_{sw}$, $N_{sp}$ with $N_f$+ $N_s$+

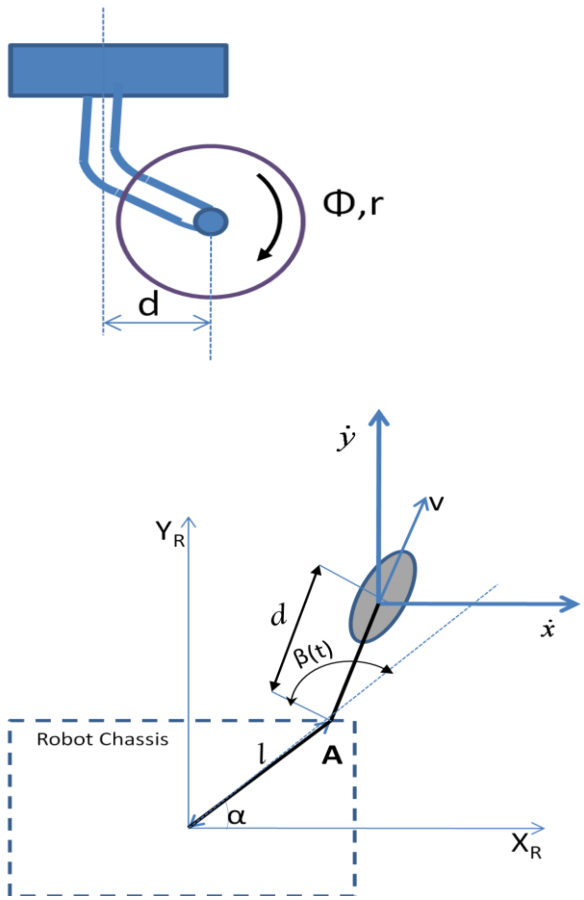


**Figure 9.** Castor wheel and its parameters.

$N_c$+ $N_{sw}$+ $N_{sp}$ = N. The configuration of the robot is fully described by the following vectors of coordinates

1. Posture coordinates : $\xi_I = [x(t) \quad y(t) \quad \theta(t)]^T$
2. Angular coordinates: $\beta_f$, $\beta_s(t)$, $\beta_c(t)$, $\beta_{sw}(t)$, and $\beta_{sp}(t)$ for the five types of wheels, respectively.
3. Rotational coordinates: $[\varphi_f(t) \quad \varphi_s(t) \quad \varphi_c(t) \quad \varphi_{sw}(t) \quad \varphi_{sp}(t)]^T$ for the rotation angles of the wheels around their horizontal axis of rotation.

The rolling constraints of all wheels can now be collected in a single expression:

$$J_1(\beta)R(\theta)\dot{\xi}_I - J_2\dot{\varphi} = 0 \quad (15)$$

This expression bears a strong resemblance to the rolling constraint of a single wheel, but substitutes matrices in lieu of single values, thus taking into account all wheels. $J_2$ is a constant diagonal matrix N × N whose entries are radii r of all standard wheels. $J_1(\beta)$ denotes a matrix with projections for all wheels to their motions along their individual wheel planes:

$$J_1(\beta) = \begin{bmatrix} J_{1f} \\ J_{1s}(\beta_s) \\ J_{1c}(\beta_c) \\ J_{1sw} \\ J_{1sp} \end{bmatrix} \quad (16)$$

where $J_{1f}$, $J_{1s}$, $J_{1c}$, $J_{1sw}$, and $J_{1sp}$ are the matrices of ($N_f$ × 3), ($N_s$ × 3), ($N_c$ × 3), ($N_{sw}$ × 3) and ($N_{sp}$ × 3), whose forms derive readily from the constraints (5), (7), (9), (11) and (13). $J_2$ is a constant (N × N) matrix whose diagonal entries are the radii of the wheels, except for the radii of the Swedish wheels which are multiplied by cosγ.

We use the same technique to collect the sliding constraints of all standard wheels into a single expression with the same structure as Equations (15) and (16):

$$C_1(\beta)R(\theta)\dot{\xi}_I + C_2\dot{\beta}_S = 0 \quad (17)$$

where $C_1(\beta) = \begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \\ C_{1c}(\beta_c) \\ C_{1sw} \\ C_{1sp} \end{bmatrix}$ and $C_2 = \begin{bmatrix} 0 \\ 0 \\ C_{2c} \\ 0 \\ 0 \end{bmatrix}$

The terms $C_{1f}$, $C_{1s}$, $C_{1c}$, $C_{1sw}$, and $C_{1sp}$ are the matrices of ($N_f$ × 3), ($N_s$ × 3), ($N_c$ × 3), ($N_{sw}$ × 3) and ($N_{sp}$ × 3), whose forms derive readily from the constraints (6), (8), (10), (12) and (14). $C_2$ is a constant (N × N) matrix whose diagonal entries are equal to d for $N_c$ of the castor wheels.

## MANEUVERABILITY OF A MOBILE ROBOT

The kinematic mobility of a robot chassis is its ability to directly move in the environment. The basic constraint limiting mobility is the rule that every wheel must satisfy its sliding constraint. In addition to instantaneous kinematic motion, a mobile robot is able to further manipulate its position, over time, by steering steerable wheels. The overall maneuverability of a robot is thus a combination of the mobility available based on the kinematic sliding constraints of the standard wheels, plus the additional freedom contributed by steering and spinning the steerable standard wheels.

### Degree of mobility

We can observe from the wheel kinematic constraints

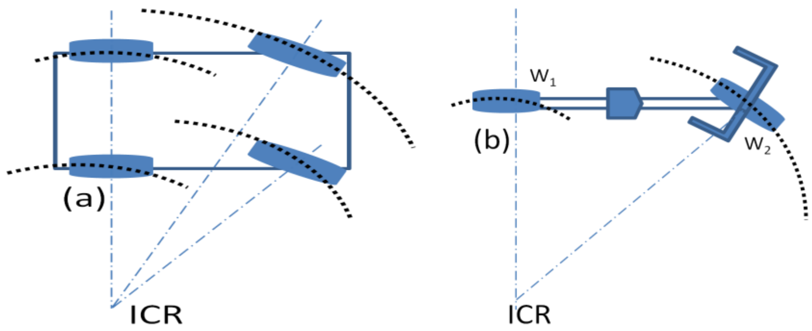


**Figure 10.** (a) Four wheel with car like Ackerman steering and (b) bicycle.

in Equations (9), (11), and (13) that the Swedish wheel, spherical wheel and castor wheel impose *no* kinematic constraints on the robot chassis, since can range freely in all of these cases owing to the internal wheel degrees of freedom. Therefore only fixed standard wheels and steerable standard wheels have impact on robot chassis kinematics and therefore require consideration when computing the robot's kinematic constraints.

Consider now the ($N_f$ + $N_S$) wheels of fixed and steered standard wheels. To avoid any lateral slip the motion vector $R(\theta)\dot{\xi}_I$ has to satisfy the following constraints:

$$C_{1f} R(\theta)\dot{\xi}_I = 0 \quad (18)$$

$$C_1(\beta_s) R(\theta)\dot{\xi}_I = 0 \quad (19)$$

And $C_1(\beta_s)=\begin{bmatrix} C_{1f} \\ C_{1s}(\beta_s) \end{bmatrix}$

Mathematically, it represents $R(\theta)\dot{\xi}_I$ must belong to the null space of the projection matrix. Null space of $C_1(\beta_s)$ is the space N such that for any vector n in N

$$C_1(\beta_s).n = 0 \quad (20)$$

The kinematic constraints [Equations (18) and (19)] can also be demonstrated geometrically using the concept of a robot's *instantaneous center of rotation* (ICR). According to this at any given instant, wheel motion along the zero motion line must be zero. In other words, the wheel must be moving instantaneously along some circle of radius such that the center of that circle is located on the zero motion line. This center point, called the instantaneous center of rotation, may lie anywhere along the zero motion line. When *R* is at infinity the wheel moves in a straight line. This ICR geometric construction demonstrates how robot mobility is a function of the number of constraints on the robot's motion, not the number of wheels.

A robot such as the Ackerman vehicle in Figure 10a can have several wheels, but must always have a single ICR. Because its zero entire motion lines meet at a single point, there is a single solution for robot motion, placing the ICR at this meet point. In figure 10b, the bicycle shown has two wheels $W_1$, and $W_2$. Each wheel contributes a constraint, or a zero motion line. Taken together, the two constraints result in a single point as the only remaining solution for the ICR.

Robot chassis kinematics is therefore a function of the set of *independent* constraints arising from all standard wheels. The mathematical interpretation of independence is related to the *rank* of a matrix. Equation (17) represents all sliding constraints imposed by the wheels of the mobile robot. Therefore $rank[C_{1s}(\beta_s)]$ is the number of independent constraints.

In general, a robot will have zero or more fixed standard wheels and zero or more steerable standard wheels. We can therefore identify the possible range of rank values for any robot: $0 \le [C_{1s}(\beta_s)] \le 3$.

Now we are ready to formally define a robot's *degree of mobility* $\delta_m$:

$$\delta_m = dimN[C_1(\beta_s)] = 3 - rank[C_1(\beta_s)] \quad (21)$$

***Examples***

$rank[C_1(\beta_s)] = 0$: If there are zero independent kinematic constraints in $C_1(\beta_s)$, then this condition can be possible. It means that the robot frame equipped with neither fixed nor steerable standard wheels, that is, $N_f$ = $N_s$ = 0.

Figure 11 represents that robot has three castor wheels and ICR can be located at any position. Therefore the degree of mobility $\delta_m = 3 - rank[C_1(\beta_s)]$ = 3

$rank[C_1(\beta_s)] = 1$: If a robot equipped with only one fixed standard wheel at the position relative to the robot's local reference frame, $C_{1s}$ is empty since there are no

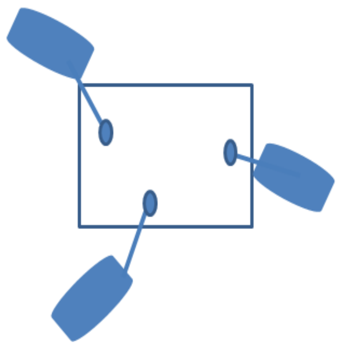

**Figure 11.** Robot having fully free motion.

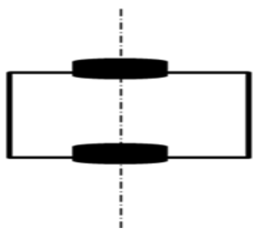

**Figure 12.** Differential-drive robot.

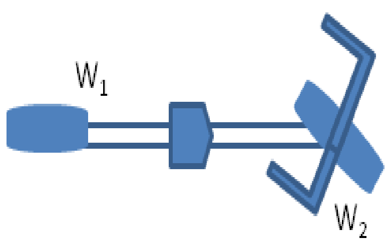


**Figure 13.** Bicycle configured robot with two fixed standard wheels.

steerable standard wheels.

Therefore $C_1(\beta_s)$ contains only $C_{1f}$. From Equation (6), we can obtain the following equation.

$$C_1(\beta_s) = C_{1f} = [\cos(\alpha + \beta) \quad \sin(\alpha + \beta) \quad l\sin\beta] \quad (22)$$

The aforementioned matrix has a rank of one since there is only one fixed standard wheel. Therefore the robot has a single independent constraint on mobility.

Consider a robot with two fixed standard wheels having specifications $l_1 = l_2, \beta_1 = \beta_2 = 0$, and $\alpha_2 = \alpha_1 + \pi$. The configuration is like a differential-drive robot as shown in Figure 12.

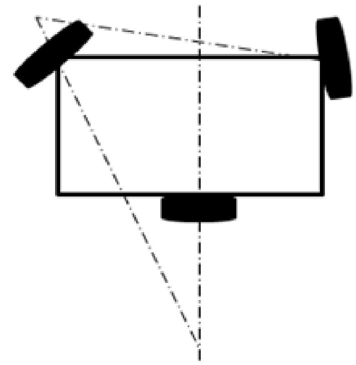

**Figure 14.** Constrained mobile robot.

Then the matrix $C_1(\beta_s)$ has two constraints but rank one. Therefore the degree of mobility $\delta_m = 3 - rank[C_1(\beta_s)] = 2$

$$C_1(\beta_s) = C_{1f} = \begin{bmatrix} \cos(\alpha_1) & \sin(\alpha_1) & 0 \\ \cos(\alpha_1 + \pi) & \sin(\alpha_1 + \pi) & 0 \end{bmatrix} = \begin{bmatrix} \cos(\alpha_1) & \sin(\alpha_1) & 0 \\ -\cos(\alpha_1) & -\sin(\alpha_1) & 0 \end{bmatrix} \quad (23)$$

$rank[C_1(\beta_s)] = 2$: Consider a robot with two fixed standard wheels configuration like bicycle with the steering locked in the forward position as shown in Figure 13.

The matrix $C_1(\beta_s)$ retains two independent constraints and has a rank of two. Therefore the degree of mobility $\delta_m = 3 - rank[C_1(\beta_s)] = 1$

Let us take $l_1 = l_2 = 1, \beta_1 = \beta_2 = {}^{\pi}/_{2}$, and $\alpha_1 = 0, \alpha_2 = \pi$

$$C_1(\beta_s) = C_{1f} = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ \cos 3\left(\frac{\pi}{2}\right) & \sin 3\left(\frac{\pi}{2}\right) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad (24)$$

$rank[C_1(\beta_s)] = 3$ : It means the robot is completely constrained in all directions and is, therefore, degenerate since motion in the plane is totally impossible. Figure 14 represents how the robot is completely constrained by considering three fixed wheels.

For this configured mobile robot the matrix $C_1(\beta_s)$ retains three independent constraints and has a rank of three. Let assume the specifications for the robot shown

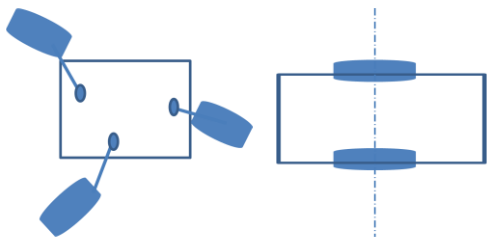

**Figure 15.** No centered orientable wheels.

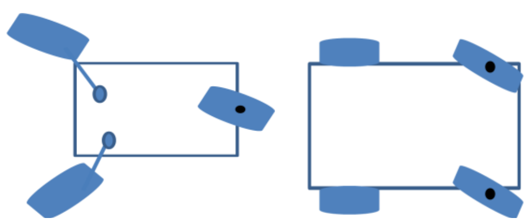

**Figure 16.** WMR with one steerable wheel and mutually dependent centered orientable wheels.

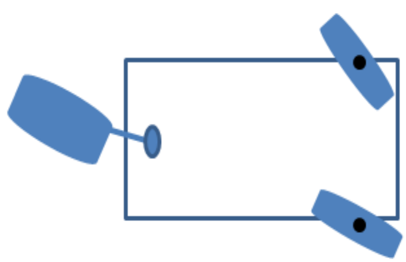

**Figure 17.** WMR with two steerable standard wheels.

in Figure 11 $l_1 = l_2 = l_3 = 1$, $\beta_1 = \beta_2 = \beta_2 = {}^{\pi}/_{2}$, and $\alpha_1 = \pi, \alpha_2 = {}^{\pi}/_{3}, \alpha_3 = {}^{-\pi}/_{4}$

$$C_1(\beta_s) = C_{1f} = \begin{bmatrix} \cos\frac{3\pi}{2} & \sin\frac{3\pi}{2} & 1 \\ \cos\frac{5\pi}{6} & \sin\frac{5\pi}{6} & 1 \\ \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad (25)$$

By observing Equation (24), it is obvious that the $rank[C_1(\beta_s)] = 3$ and its degree of mobility $(\delta_m)$ is 3-$rank[C_1(\beta_s)]$ = 0.

## Degree of steerability

AWMR can be equipped with the number of centered orientable wheels in order to steer the robot. This impact of steering is indirect since the robot must move for the change in steering angle of the steerable standard wheel. The degree of steerability is defined:

$$\delta_s = \text{rank}\left[C_{1s}(\beta_s)\right] \quad (26)$$

An increase in the rank of $\left[C_{1s}(\beta_s)\right]$ implies more degrees of steering freedom and thus greater eventual maneuverability. The range of $\delta_s$ is given by $0 \leq \delta_s \leq 2$. Since $\left[C_1(\beta_s)\right]$includes$\left[C_{1s}(\beta_s)\right]$, a steerable standard wheel can both decrease mobility and increase steerability:

1. Its particular orientation at any instant imposes a kinematic constraint.
2. Its ability to change that orientation can lead to additional trajectories.

### *Examples*

1. *For $\delta_s$= 0:* The robot has no centered orientable wheels, $N_s$ = 0 as shown in Figure 15. Therefore its degree of steerability is 0.
2. *For $\delta_s$= 1:* Consider a robot has one centered orientable wheel, that is, $N_s$ = 1 or two mutually dependent centered orientable wheels as shown in Figure 16.

$$C_{1s}(\beta_s) = \left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right] \quad (27)$$

Therefore rank $\left[C_{1s}(\beta_s)\right]$ is one and its Degree of steerability $\delta_s$= *1.*

3. *For $\delta_s$= 2:* consider a robot has one centered orientable wheel, that is, $N_s$ = 2 and those are independent on each other as shown in Figure 17.

$$C_{1s}(\beta_s) = \begin{bmatrix} \cos(\alpha_1+\beta_1) & \sin(\alpha_1+\beta_1) & l\sin\beta_1 \\ \cos(\alpha_2+\beta_2) & \sin(\alpha_2+\beta_2) & l\sin\beta_2 \end{bmatrix} \quad (28)$$

Therefore the rank $\left[C_{1s}(\beta_s)\right]$ is two and it's degree of steerability $\delta_s$= *2.*

## Maneuverability of WMR

The overall (Degrees of Freedom) DOF that a robot can manipulate is called the degree of maneuver-ability$(\delta_M)$. It can be defined in terms of mobility and steerability. Thus

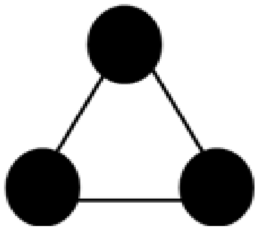

**Figure 18.** Three wheeled omnidirectional mobile robot.

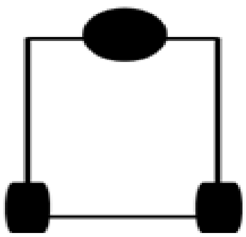

Figure 19. Omnidirectional three WMR.

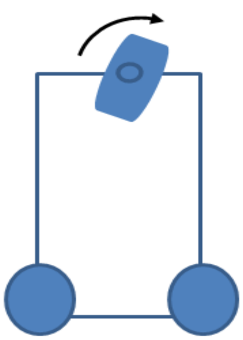

**Figure 20.** Three wheeled omni-steer mobile robot.

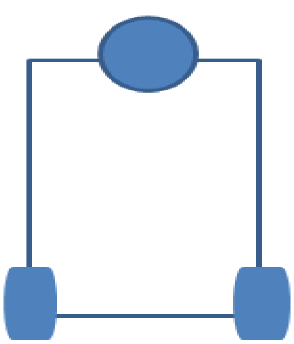

**Figure 19.** Omnidirectional three WMR.

the maneuverability consists of the degrees of freedom that the robot manipulates directly through wheel velocity and the degrees of freedom that it indirectly manipulates by changing the steering configuration and moving.

$$\therefore \delta_M = \delta_m + \delta_s \qquad (29)$$

1. *Omni directional:* Figure 18 represents an omni-directional mobile robot having three wheels other than fixed and steered standard wheels. For this type of robot, $rank[C_{1s}(\beta_s)]$ and $rank[C_1(\beta_s)]$ is zero since it has no steerable standard wheels and fixed standard wheels.

This results in the degree of mobility $\delta_m = 3$ and the degree of steerability $\delta_s = 0$

∴ The degree of maneuverability, $\delta_{M)} = \delta_m + \delta_s = 3$

*2. Differential*: Figure 19 represents a differential WMR having two wheels are fixed and other is except fixed and steered standard wheels. For this type of robot $rank[C_{1s}(\beta_s)]$ is zero and $rank[C_1(\beta_s)]$ is one since it has no steerable standard wheels.

This results the degree of mobility $\delta_m = 2$ and the degree of steerability $\delta_s = 0$

∴ The degree of maneuverability, $\delta_M = \delta_m + \delta_s = 2$

3. *Omni-Steer:* Figure 20 represents three wheeled omni-steer mobile robot having two wheels are other than fixed and steered standard wheels and one steered standard wheel. For this type of robot $rank[C_{1s}(\beta_s)]$ is one and $rank[C_1(\beta_s)]$ is one since it has one steerable standard wheel.

This results in the degree of mobility $\delta_m = 2$ and the degree of steerability $\delta_s = 1$

∴ The degree of maneuverability, $\delta_M = \delta_m + \delta_s = 3$

4. *Tricycle:* Figure 21 represents tricycle mobile robot having two fixed wheels and one steered standard wheel. Steering and power are provided through the front wheel. For this type of robot $rank[C_{1s}(\beta_s)]$ is one and $rank[C_1(\beta_s)]$ is two since it has one steerable standard wheel.

This results the degree of mobility $\delta_m = 1$ and the degree of steerability $\delta_s = 1$

∴ The degree of maneuverability, $\delta_M = \delta_m + \delta_s = 1$.

*5. Two steer:* Figure 22 represents two steer three wheeled mobile robot having two steered standard wheels and one omnidirectional wheel. For this type of robot $rank[C_{1s}(\beta_s)]$ is two and $rank[C_1(\beta_s)]$ is two since it has two steerable standard wheels.

This results the degree of mobility, $\delta_m = 1$ and the degree of steerability $\delta_s = 1$

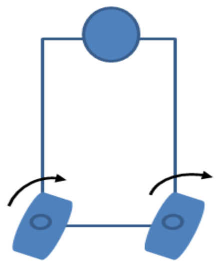

**Figure 22.** Two steer mobile robot.

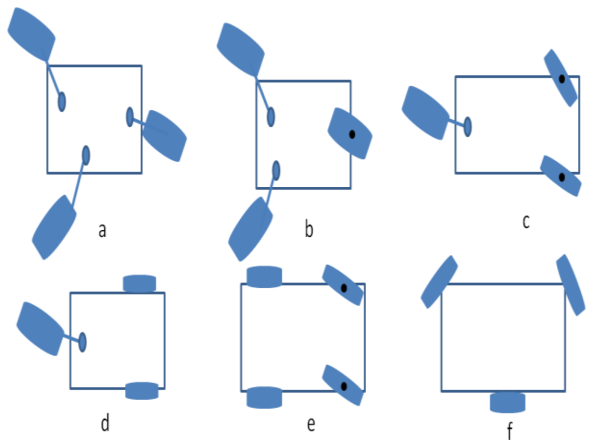


**Figure 23.** WMRs with various configured wheels.

**Table 1.** Maneuverability of various configured wheels for Figure 23.

| Figure | $\delta_m$ | $\delta_s$ | $\delta_M$ |
|---|---|---|---|
| a (neither fixed wheels nor centered orientable wheels) | 3 | 0 | 3 |
| b (one steerable standard wheel and two castor wheels) | 2 | 1 | 3 |
| c (two independent steerable standard and one castor wheels) | 1 | 2 | 3 |
| d (two dependent fixed and one castor wheels) | 2 | 0 | 2 |
| e (two dependent fixed and centered orientable wheels) | 1 | 1 | 2 |
| f (three fixed wheels) | 0 | 0 | 0 |

∴ The degree of maneuverability$(\delta_M) = \delta_m + \delta_s = 1$

6. *Maneuverability of various configured WMR:* Consider various WMR configurations as shown in Figure 23 having three or four wheels with arbitrary orientations with respect to its wheel plane. The maneuverability of these WMRs can be observed in Table 1.

## Conclusion

Kinematic model for various wheeled mobile robots have been developed. The possible wheel configurations have been categorized into five types for all WMRs according to their mobility restriction induced by the kinematic constraints. Posture kinematic model for a mobile robot has been derived, which is adequate to describe the global motion of the robot. The results obtained from these kinematic models have been applied to various types of WMRs in order to get their degree of mobility and its maneuverability. The present research on kinematic analysis can be applied in various fields such as the automotive industry, especially to all wheel drive electric vehicles. Since the analysis has been carried out under ideal considerations, the frictional effects induced between the wheels of the robot and surface is neglected. As a future work, it is necessary to develop kinematic models for WMRs by considering practical conditions and it is required to develop the control algorithms and the motion experiments for real application.

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