

Full Length Research Paper

Acceleration motion of geometric and spherical particles in two dimensions and implications in design of continuous sedimentation rectangular tanks

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The design procedure outlined by Camp (1946) for design of continuous gravity sedimentation tank was revisited. The viscous effects of flowing fluid were included in the model by development of the velocity profile of the fluid in the horizontal direction. The transient motion of the spherical and geometric particles unhindered prior to reaching terminal settling velocity, was simulated using a desktop computer. The governing equations in two dimensions, vertical and horizontal were written in terms of velocity of the particle and the drag coefficient in transient motion was assumed to be of the same functional form as that obtained from empirical observations at steady state. The five constant expressions of Turton and Levenspiel (1989) was used and the trajectory of the particle was obtained relative to the motion of the fluid by use of fifth order Runge-Kutta method of numerical integration. As the density of the particle and size of the particle increases, the acceleration zone of the particles increased in size. Deeper tanks have to be constructed for such systems. The geometric particles reached their terminal settling velocities sooner compared with the spherical particles. The pressure drop, throughput and separation efficiency trade-offs are discussed.

Key words: Continuous gravity sedimentation, trajectory of particle, drag coefficient correlations, transition flow, fifth order Runge-Kutta method.

INTRODUCTION

Sedimentation is a solid-liquid separation method using gravity settling to remove suspended solids, by Reynolds and Richards (1996). Sludge in surface waters are allowed to settle in sedimentation tanks. In an iron and manganese removal plant this method is used. Coe and Clevenger (1916) classified types of settling into four types. During type I, settling particles sink relative to the fluid in motion free of interference from other particles. Camp's (1936) theory (in Davis, 2011) was based on the principle that in order for a particle to be removed from the flowing stream of fluid, the particle must have a terminal settling velocity, v_{px}^t , great enough so that it reaches the bottom of the tank during the residence time, θ of the fluid in the tank. Thus:

$$v_{px}^t = \frac{H}{\theta} \quad (1)$$

The residence time of the fluid in tank can be estimated from the discharge rate Q and volume of the tank:

$$\theta = \frac{HWL}{Q} \quad (2)$$

Where, H , W and L and the height, width and length of the rectangular sedimentation tank and Q is the discharge rate of water in $m^3.s^{-1}$. Combining Equations (1 and 2):

$$v_{px}^t = \frac{Q}{WL} = \frac{Q}{A_s} \quad (3)$$

Camp (1946) suggested that the overflow rate v_0 be set at v_{px}^t . The implications of this design is that the particle removal efficiency is independent of the depth in the

tank, H and the residence time of the fluid in the tank, θ . Further, the calculation of terminal settling velocity of the particle is assumed to obey the Stokes law (1822). Stokes law is applicable only when the Reynolds' number of fluid around the particle is small, that is $Re < 200$. In this study, a method is developed to include the effect of the height of the tank in the sedimentation design. This is proposed to be included by use of the viscous flow of water in the horizontal direction. A parabolic velocity profile with a maximum velocity at the top of the tank especially when the top lid is open and zero velocity at the sludge zone is derived from the Hagen (1839) and Poiseuille (1841) law, for circular conduits applied to a rectangular chamber.

Further, the findings from an earlier study by Renganathan et al. (1989) for acceleration motion of geometric and spherical particles in a stationary Newtonian fluid is extended to two dimensions, that is, x and y in this study. Their study revealed that the vertical distance travelled by a spherical particle prior to reaching 90% of its terminal settling velocity was found from numerical simulations to be:

$$X = \frac{x\gamma}{d_p} = \frac{1.27}{C_{Dt}^{0.93}} \quad (4)$$

For an iron particle size of 2.5 cm with a density of 7800 kg.m⁻³ this distance would be about 49.9 cm. This is about 25 to 50% of the depths used in design of settling tanks. The Reynolds number and drag coefficient at terminal settling velocity can be calculated using trial and error by Equation (5). This was found to be a Reynolds number of 54,250 and a drag coefficient of 0.47. For such cases, the assumption that the overflow velocity of the sedimentation tank is independent of the height may be a poor assumption. Numerical simulations are conducted on the desktop computer using the fifth order Runge-Kutta method, using MS Excel 2007 for windows software. The results from the simulation are used to modify the design procedure suggested by Camp (1946). Since the work of Camp (1946, 1952) much has been discovered about steady state drag coefficients.

Raleigh (1883) was one of the first to express the drag coefficient as a function of the Reynolds number. Clift et al. (1978) have provided a review of the available empirical correlations for C_{Dt} vs. Reynolds number of the fluid at terminal settling velocity of the particle. Chabra (1993) has presented an analytical solution for acceleration motion of a spherical particle in an unbounded Newtonian fluid. The drag correlation used was of the form:

$$C_D = \left(\frac{A}{Re^{1/2}} + B \right)^2 \quad (4a)$$

They use a $Re = p^2$ substitution. Turton and Levenspiel (1989) provided a 5 constant expression for C_{Dt} vs. Reynolds' number. This expression is capable of showing a minimum C_D . As a result, this expression was used in this study compared with relations such as in Equation (4a) etc. This expression is given as follows:

$$C_{Dt} = \frac{24}{Re_t} \left(1 + 0.173 Re_t^{0.657} \right) + \frac{0.413}{1 + 16300 Re_t^{-1.09}} \quad (5)$$

This expression was found to correlate well with the experimental data over a wider range of Reynolds' number from 1 to 200,000. This expression reverts to the Stokes' expression at low Reynolds' number. This expression was used in the numerical simulation study of the trajectory of a particle in a continuous rectangular sedimentation tank. Sphericities of geometric particles such as disks, isometric particles were included in the development of Equation (5) by Haider and Levenspiel (1989).

Very little discussion in the literature is available on the two dimensional trajectory of the particle relative to the moving fluid. Lapple and Shepherd (1940) wrote equations for calculating the paths taken by bodies for particles undergoing accelerated motion, taking into account the effect of fluid friction. Since the work of Lapple and Shepherd advances in the description of the drag coefficient correlation as a function of Reynolds number have been made. An expression such as Equation (5) is used in the simulations in this study. Further, a continuous sedimentation tank with viscous Newtonian flow of fluid such as water is considered in this study.

THEORY AND SIMULATION

Consider a proptotypical horizontal ideal sedimentation tank with a sectional view as shown in Figure 1. Water is allowed to flow in a continuous manner at a certain overflow rate at the weir near the outlet of the tank. The rectangular tank has dimensions of L , W and H as its length, width and height. The particles have to be captured in the sludge zone during the residence time in the tank, θ . It is that the fluid motion in x direction is negligible. All flow from inlet to outlet happens in the ' y ' direction only.

Fluid

A momentum balance on a slice $W\Delta x\Delta y$ of fluid (Bird et al., 2007) at steady state is;

$$W\Delta x(P_y - P_{y+\Delta y}) = W\Delta y(\tau_{xy} \Big|_x - \tau_{xy} \Big|_{x+\Delta x}) \quad (6)$$

In the limits, when Δx and Δy tend to zero, Equation (6) becomes:

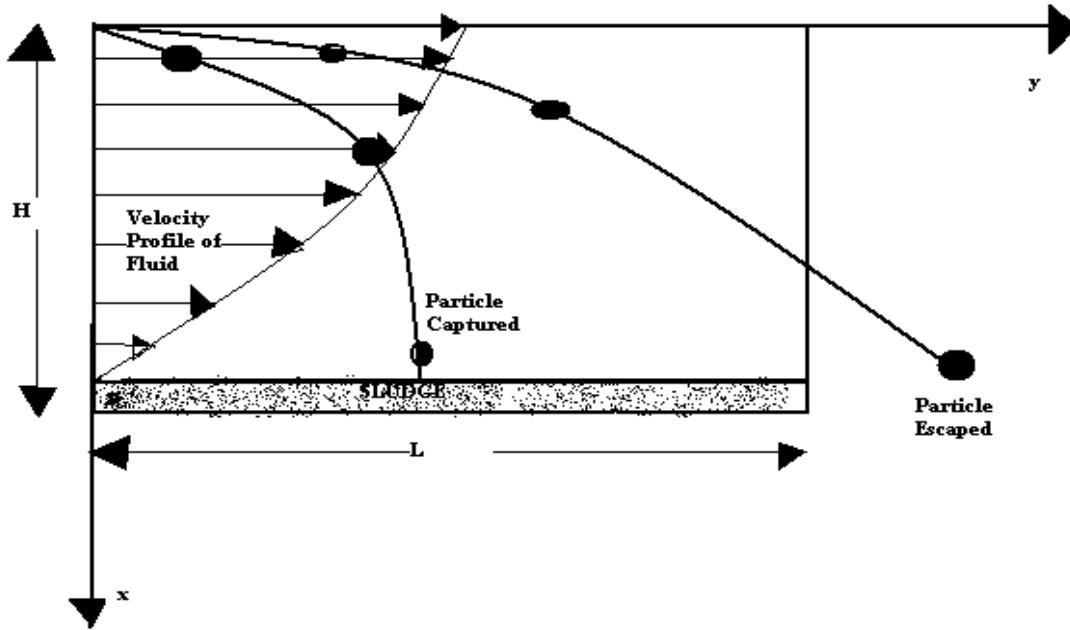


Figure 1. Horizontal ideal sedimentation tank.

$$\left(\frac{\partial P}{\partial y}\right) = \left(\frac{\partial \tau_{xy}}{\partial x}\right) \quad (7)$$

Linear pressure drop is assumed and:

$$\left(\frac{\partial P}{\partial y}\right) = \left(\frac{\Delta P}{L}\right) = \left(\frac{\partial \tau_{xy}}{\partial x}\right) \quad (8)$$

Equation (8) is integrated to yield:

$$\tau_{xy} = \left(\frac{\Delta P}{L}\right)x + c_1 \quad (9)$$

Velocity profile of the fluid is maximum at $x = 0$. Hence c_1 in Equation (9) is zero. Writing the Newton's Law of viscosity, Equation (9) would become:

$$-\mu \left(\frac{\partial v_y}{\partial x}\right) = \left(\frac{\Delta P}{L}\right)x \quad (10)$$

Integration of Equation (10) yields:

$$v_y = -\left(\frac{\Delta P x^2}{2\mu L}\right) + c_2 \quad (11)$$

Velocity of the fluid, $v_y = 0$ at the sludge zone. c_2 is evaluated and:

$$v_y = \left(\frac{\Delta P H^2}{2\mu L}\right) \left(1 - \left(\frac{x}{H}\right)^2\right) \quad (12)$$

The average velocity of the fluid $\langle v_y \rangle$ can be obtained by integration of Equation (12) between the definite limits of 0 and H and shown to be:

$$\langle v_y \rangle = \left(\frac{\Delta P H^2}{3\mu L}\right) \quad (13)$$

The implications of the average horizontal velocity of the fluid can be seen in the residence time expression. Thus, Equation (2) for residence time, θ , from Camp (1946) need be modified as follows:

$$\theta = \frac{HWL}{\langle v_y \rangle WH} = \frac{L}{\langle v_y \rangle} = \frac{3\mu L^2}{\Delta P H^2} \quad (14)$$

Single particle – vertical direction

Consider the relative motion of a spherical particle that is settling towards the sludge zone in the vertical direction. A force balance on the particle can be written as a resultant of the gravity force, Archimedes buoyancy force and drag force. Thus;

$$m_p \left(\frac{\partial v_{xp}}{\partial t}\right) = (\rho_s - \rho)V_p g - \frac{C_D \rho v_{xf}^2 A_p}{2} \quad (14)$$

The fluid velocity, v_{xf} surrounding the particle with a vertical velocity of v_{xp} will be equal to each other. Defining the following dimensionless variables:

$$\gamma = \frac{\rho}{\rho_s}; Y = \frac{v_{px}}{v_{px}^t}; X = \frac{y}{d_p}; \quad (15)$$

Where the terminal settling velocity of the particle in x direction is given by v_{px}^t . It was shown by Renganathan et al. (1989) that Equation (14) can be written in dimensionless form, after expression of acceleration as a function of velocity as “ $adx = v_x dv_x$ ” as follows:

$$\int dX = \int \frac{YdY}{0.75(C_{Dt} - Y^2 C_D)} \quad (16)$$

Where C_{Dt} is the drag coefficient at terminal settling velocity of the particle. The expression for C_{Dt} can be seen to be:

$$C_{Dt} = \frac{4gd_p(\rho_s - \rho)}{3\rho v_{px}^t{}^2} \quad (17)$$

Equation (16) was simulated on the desktop computer. This form of the equation was found to be less error-prone compared to equations in terms of time. The equations in time domain also are an order higher, that is, second order equation. The drag coefficient C_D in the accelerating regime was assumed to be of the same functional form as found at steady state by Turton and Levenspiel and given by Equation (5). The numerical integration method selected, was the fifth order Runge-Kutte method, using MS Excel 2007 for windows. The Butcher’s (1964) method as described in Chapra and Canale (2006) was used. The recurrence relations used are as follows:

$$y_{i+1} = y_i + \frac{h}{90}(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6) \quad (18)$$

$$k_1 = f(x_i, y_i) \quad (19)$$

$$k_2 = f(x_i+0.25h, y_i+0.125k_1h+0.125k_2h) \quad (20)$$

$$k_3 = f(x_i+0.25h, y_i-0.5k_2h+k_3h) \quad (21)$$

$$k_4 = f(x_i+0.5h, y_i-0.5k_2h+k_3h) \quad (22)$$

$$k_5 = f(x_i+0.75h, y_i+3/16k_1h+9/16k_4h) \quad (23)$$

$$k_6 = f(x_i+h, y_i-3/7k_1h+2/7k_2h+12/7k_3h-12/7k_4h+8/7k_5h) \quad (24)$$

where h is the increment in the independent variable, Y. The simulations were run till the particle reached 95% of its terminal settling velocity.

Single particle – vertical direction

The governing equations for relative motion of the particle in the y direction can be written as follows:

$$m_p \left(\frac{\partial v_{yp}}{\partial t} \right) = \frac{\Delta P x A_p}{L} - \frac{C_D \rho v_{yf}^2 A_p}{2} \quad (25)$$

The fluid velocity, v_{yf} surrounding the particle with a vertical velocity of v_{yp} will be equal to each other. Defining the following dimensionless variables:

$$\gamma = \frac{\rho}{\rho_s}; Z = \frac{v_{py}}{v_{py}^t}; \xi = \frac{y}{d_p}; \quad (26)$$

Where the terminal settling velocity of the particle in y direction is given by v_{py}^t . Equation (25) can be written in dimensionless form after expression of acceleration as a function of velocity as “ $ady = v_y dv_y$ ” as follows:

$$\int d\xi = \int \frac{ZdZ}{0.75(C_{Dt} - Z^2 C_D)} \quad (27)$$

Where C_{Dt} is the drag coefficient at terminal settling velocity of the particle. The expression for C_{Dt} can be seen to be:

$$C_{Dt}^y = \frac{2\Delta P x}{3v_{py}^t{}^2 L \rho_s} \quad (28)$$

Equation (16) was simulated using the fifth order Runge-Kutte method, using MS Excel 2007 for windows. The Butcher’s (1964) method as described in Chapra and Canale (2006) was used in the simulations.

RESULTS AND DISCUSSION

A 2.5 cm, iron spherical particle was considered. A million gallons per day of water flows through a sedimentation tank with a weir area of 0.001 m². The pressure drop considered is about 50 N.m². The trajectory of the 2.5 cm iron particle obtained from the computer simulations is shown in Figure 2. It can be seen that the 2.5 cm iron particle reaches the sludge zone, the particle has reached 66% of its terminal settling velocity that is, the particle is still accelerating. It has travelled 75 cm in the vertical direction and 28 cm in the horizontal direction. The sedimentation tank design has to keep the acceleration zone a smaller portion of the height of the tank. In this case, the sedimentation tank has to be made deeper. Only then will the design overflow velocity based on the terminal settling velocity of the particle, result in the estimated separation efficiency. Otherwise, the separation efficiency realized will be lower. The particle is subjected to an increasing force as it travels in the y direction. This is because of the velocity profile as shown

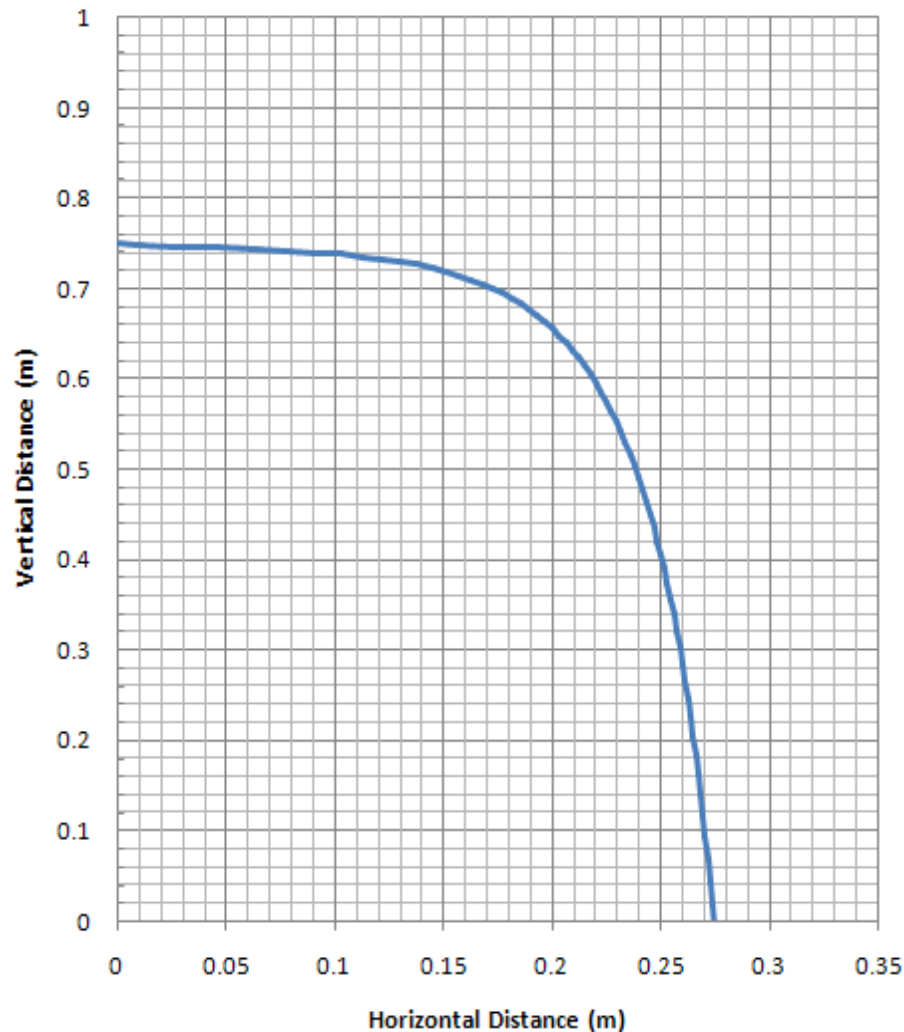


Figure 2. Trajectory of 2.5 cm iron particle in horizontal sedimentation tank.

in Figure 1. The drag force will oppose the motion of the particle in the y direction as described in Equation (25) and rise to meet the applied kinetic force at the terminal settling velocity of the particle in the y direction.

Recent designs of horizontal gravity sedimentation tanks provide a slope to minimize motion of particles in ' y ' direction in the sludge zone. In Figure 3 is shown a simulation trial at higher pressure drop ($\Delta P=100 \text{ N.m}^{-2}$) and a deeper sedimentation tank at $H = 2 \text{ m}$. The particle arriving at the sludge zone has reached 93% of the terminal settling velocity of the particle. In Figure 4 is shown a simulation trial of a sand particle with a $\rho_s = 2,600 \text{ kg.m}^{-3}$, particle size, $d_p = 2.5 \text{ cm}$ and a height of sedimentation tank of 0.75 m . The trajectory of the accelerating sand particle is compared with the trajectory of the same particle, should the particle have attained terminal settling velocities rapidly as assumed in Camp (1946). The trajectory of the particle is curvilinear and not rectilinear as assumed in Camp's theory (Figure 4).

Conclusions

The procedure outlined by Camp (1946) to design gravity sedimentation tank has been improved upon. The Drag correlations with Reynolds number for sphere and geometric particles are used to simulate the two dimensional trajectory of the sedimenting particle. The governing equations of motion of the spherical particle are made dimensionless. The acceleration term is rewritten in terms of velocity. This results in an order reduction. The trajectory of the particle is recovered from computer simulations of Equations (16) and (27) using the fifth order Runge-Kutta integration method. The sedimentation tank needs to be made deeper, so that the acceleration zone of the particles prior to reaching terminal settling velocity is a smaller portion of the overall height of the tank. Viscous effects of the fluid are now included in the design procedure. The overflow rate is no longer independent of the height of the tank.

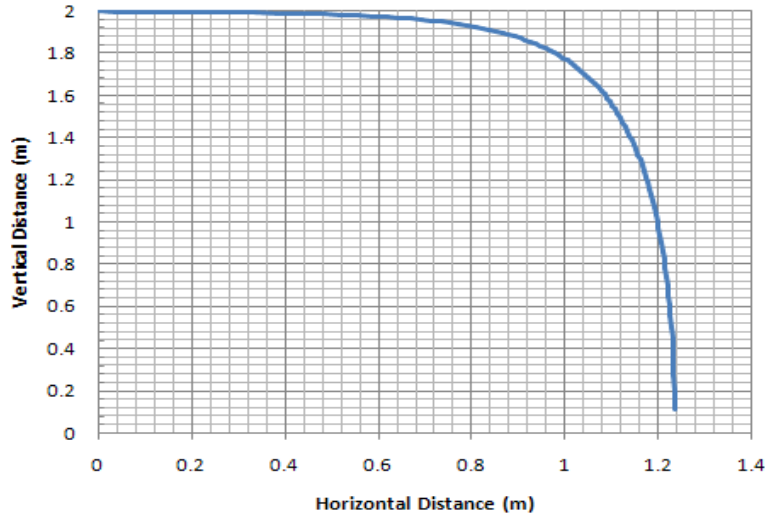


Figure 3. Trajectory of a iron particle at higher pressure drop and deeper tank.

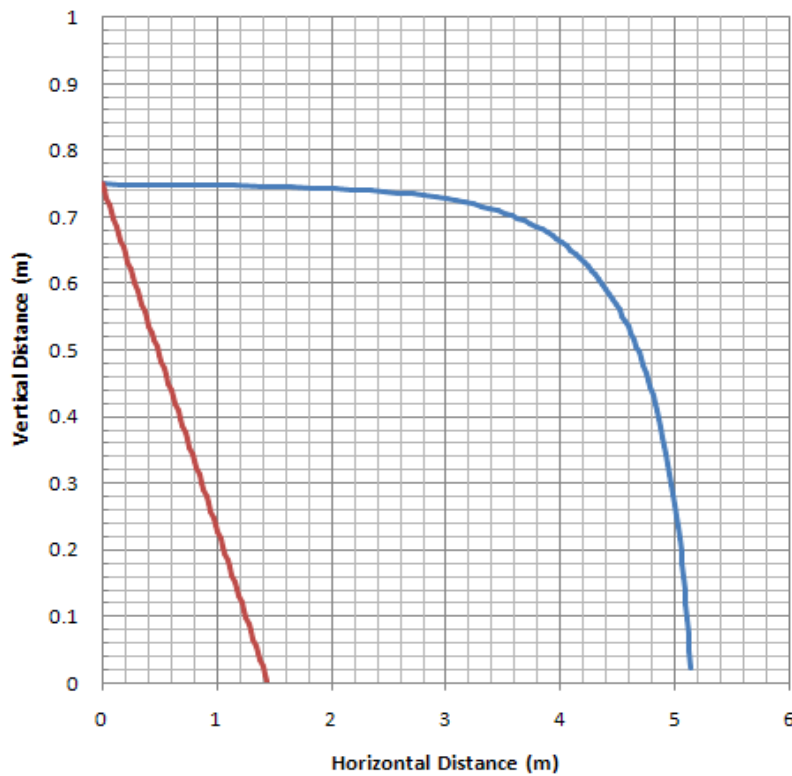


Figure 4. Trajectory of sand particle with a $d_p = 2.5$ cm.

The parabolic velocity profile of the fluid is incorporated into the mathematical model. The residence time and the average velocity of the fluid in the tank are related to each other. The higher the pressure drop in the tank the higher would be the terminal settling velocity of the

particle in the y direction. This is a trade-off between separation efficiency, throughput and pressure drop across the tank. The shape of the trajectory is a plateau in the initial phase and a sharper fall later. It is monotonic and has linear asymptotes. The trajectory is curvilinear. It

takes longer to attain the terminal settling velocity in the vertical direction compared with that in the horizontal direction at smaller pressure drops. The pressure drop can be increased so that the distance travelled to terminal settling velocity is the same in x and y directions. Expression for terminal settling velocity of the particle in y direction is given by Equation (28). The acceleration zone was found to be shorter for geometric particles. The C_D vs. Reynolds' number correlation developed by Haider and Levenspiel (1989) was used during these simulations. Upon reaching terminal settling velocities in both directions the trajectory of the particle becomes linear with curvature given by ratio of the terminal settling velocity of the particle in vertical direction to that in the horizontal direction. The trajectory of the accelerating particles were found to be curvilinear and not rectilinear as assumed in Camp's (1946) theory.

Nomenclature

A_p , Projected area spherical particle (m^2); A_s , cross-sectional area of rectangular tank; $A_s = WH$; C_D , drag coefficient (transient); C_{Dt} , drag coefficient at terminal settling velocity of particle; d_p , particle size of sphere (m); g , acceleration due to gravity ($m.s^{-2}$); h , step size used in numerical integration; H , height of the sedimentation tank (m); k_i , weighting factors used in fifth order Runge-Kutta method; L , length of the sedimentation tank (m); m_p , mass of particle (kg); ΔP , pressure DROP ($N.m^{-2}$); Q , discharge rate ($m^3.s^{-1}$); Re , Reynolds number,

$$Re = \frac{\rho v d_p}{\mu} \quad t, \text{time (s)}; v_{px}^t, \text{terminal settling velocity of}$$

the particle in x direction (m/s); v_{py}^t , terminal settling velocity of the particle in y direction (m/s); v_y , velocity of the fluid in the y direction as a function of x; $\langle v_y \rangle$, average velocity of fluid in y direction (m/s); V_p , volume of particle (m^3); W , width of the sedimentation tank (m); x , vertical distance (m); X , dimensionless vertical distance

$$\text{travelled by particle, } X = \frac{x\gamma}{d_p} \quad y, \text{horizontal distance (m);}$$

Y , dimensionless velocity of particle in vertical direction,

$$Y = \frac{v_{px}}{v_{px}^t} \quad Z, \text{dimensionless velocity of particle in}$$

$$\text{horizontal direction, } Z = \frac{v_{py}}{v_{py}^t} .$$

Greek: γ , Density ratio (ρ/ρ_s); μ , viscosity of water ($kg.m^{-1}s^{-1}$); ρ , density of fluid ($kg.m^{-3}$); ρ_s , density of particle ($kg.m^{-3}$); τ_{xy} , shear stress ($N.m^{-2}$); θ , residence time (s); ξ , dimensionless horizontal distance travelled,

$$\xi = \frac{y\gamma}{d_p} .$$

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